Neutrino Models: Where is the Landscape?

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【IHEP Beijing】

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Disclaimer

♦ In 1971, Paul A.M. Dirac made the following remarks in his J. Robert Oppenheimer Prize Lecture:

> One finds that it is really remarkable how unwilling people were to postulate a new particle. This applies to both theoretical and experimental workers. It seems that they would look for an explanation rather than postulate a new particle. The climate has completely changed since the early days. People are only too keen to publish evidence for a new particle, whether this evidence comes from experiment or from some ill-established theoretical idea.

Pierre Ramond (2005): written more than thirty years ago, this comment has gained even more relevance today, when infinite towers of new particles are shamelessly proposed to explain the slightest experimental discrepancies!

♦ This talk will avoid too model-dependent arguments and exercises, no matter how they are popular. I apologize for missing many important works due to the limited scope of my knowledge. But I will try to present something nontrivial.

Lectures by
Eung Jn Chun
Un-ki Yang ...

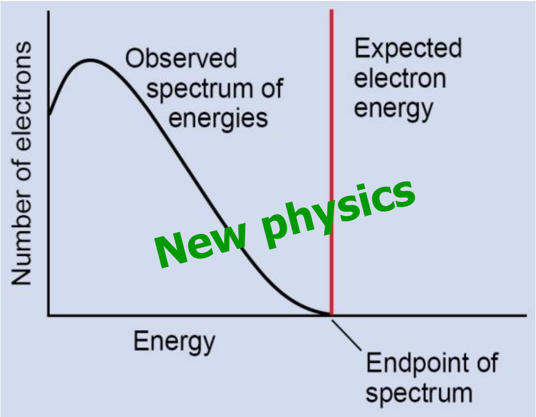
OUTLINE

- Why neutrinos and why massless?
- Why Majorana and why not Dirac?
- Possible ways to test the seesaw?
- There is a flavor symmetry behind
- Inconclusive remarks

Worse than just an energy crisis?

♦ **An unexpected continuous energy spectrum of outgoing electrons in a presumable two-body beta decay was observed (J. Chadwick 1914, C. Ellis 1920~1927), posing several challenges.**

$$n \to p + e^-$$



But neutron was not discovered until 1932.

Here is just for illustration

Y **N**

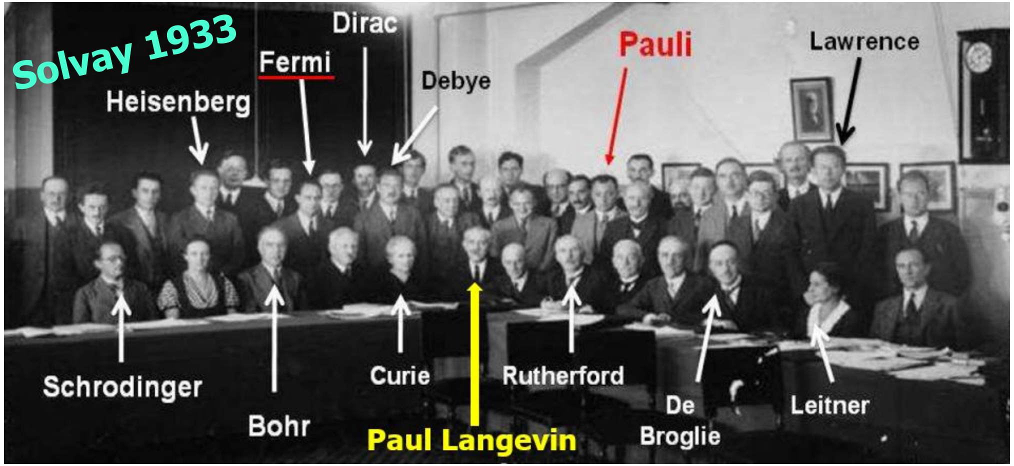
- **The law of energy conservation violated?**
- **Lepton number conservation violated?**
- **The law of angular momentum conservation violated!** **(½ ⊕ ½ = 1, 0)**

Pauli on the right side of history

♦ It was W. Pauli, father of the Pauli exclusion principle (1925), who killed the three birds with one stone —— a light, neutral and spin-half particle (1930), the electron antineutrino.

$$n \to p + e^- + \overline{\nu}_e$$

How to quickly fix the location of Mr Langevin in an old photo of the Solvays?

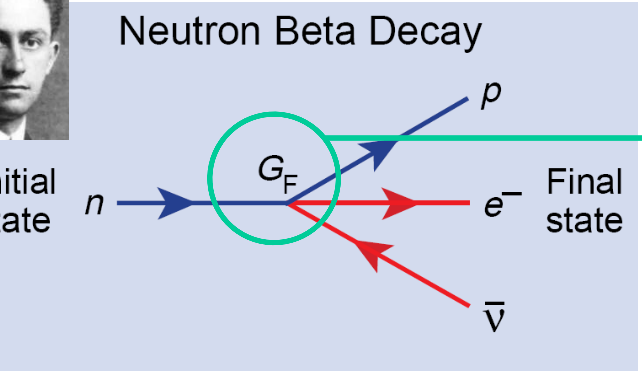


Nobel Prize
W.H. 1932
P.D. 1933
E.F. 1938
W.P. 1945

- He published his idea in an open letter (1930)
- He sold his idea to E. Fermi in Solvay Congress 1933
- He was unhappy with himself when he was old (C.N. Yang 1986)

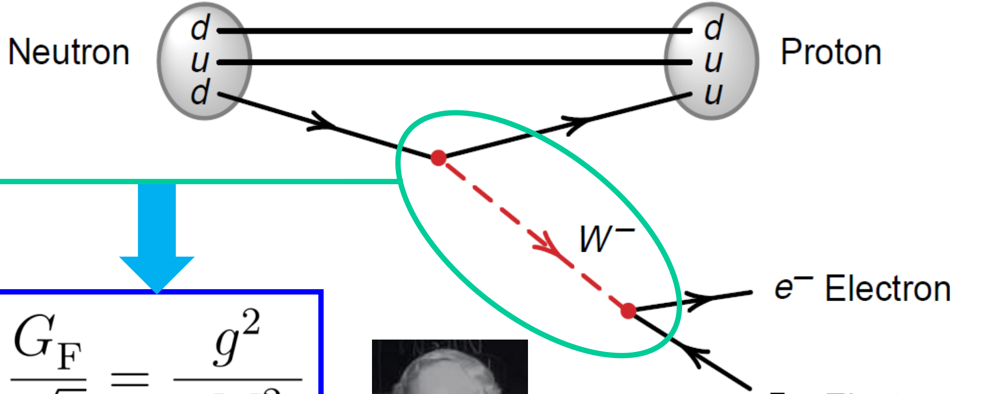
From Fermi's EFT to Weinberg's SM

♦ **E. Fermi established the β-decay EFT (1933/1934), a great step towards S. Weinberg's SM (1967)**



$$G_F(\overline{\psi}_p\,\gamma_\mu\,\psi_n)(\overline{\psi}_e\,\gamma^\mu\gamma_5\,\psi_\nu)$$

V **A**



$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

gauge seesaw

Fermi coupling constant

$$G_F \simeq 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$$

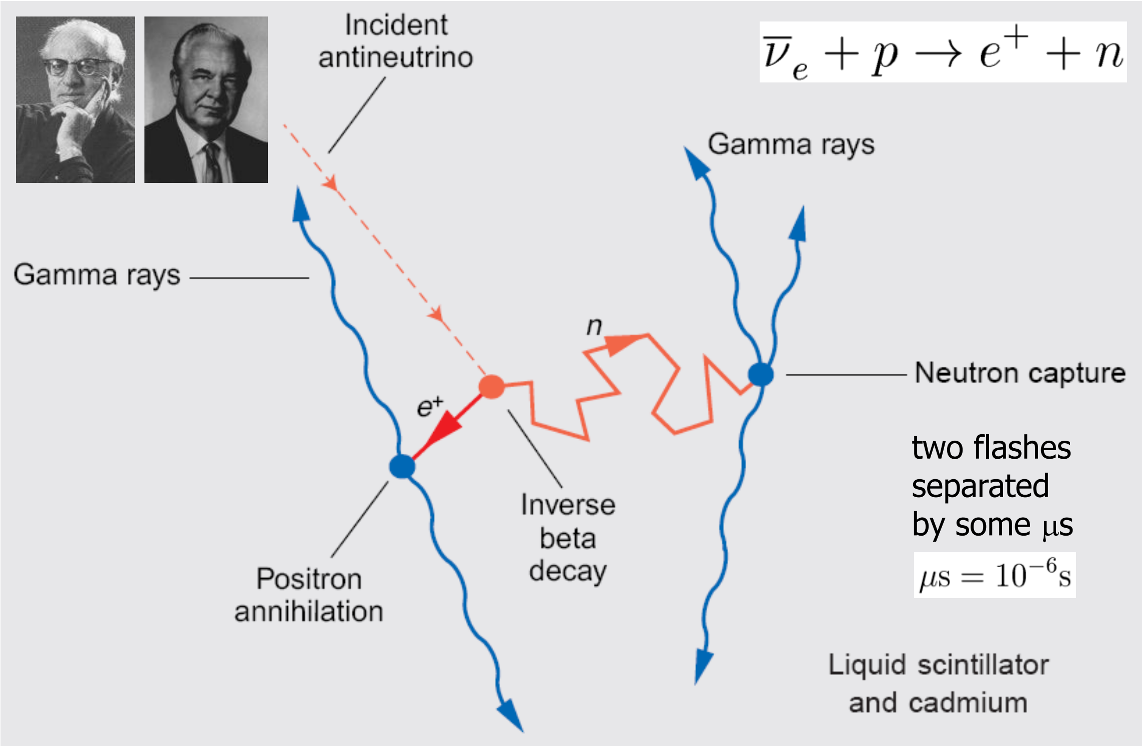
Weak interaction coupling constant

$g \simeq 0.65$ **vs** $M_W \simeq 80.4$ GeV

A good lesson: some effective quantities at low energies are very likely to originate from new heavy degrees of freedom in a more fundamental theory at much higher energy scales.

Fermi's EFT worked well

♦ F. Reines and C. Cowan discovered reactor antineutrinos (1956). But it's already too late for Pauli.



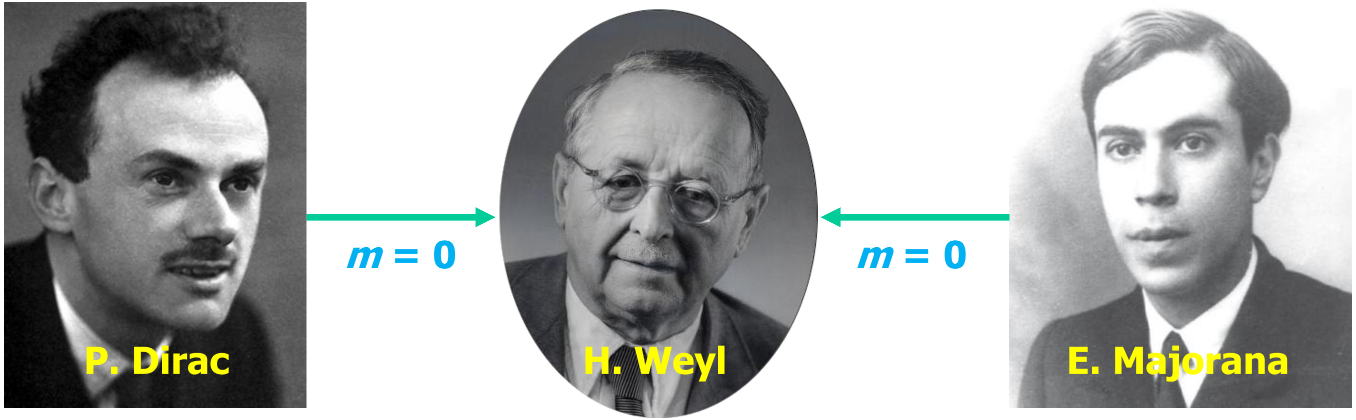
"I have already introduced one hypothetical massless particle, and I had no nerve to introduce more." (Pauli's regret in 1954)

- Parity violation: TH (1956) by T.D. Lee, C.N. Yang
- Parity violation: EX (1957) by C.S. Wu / L. Lederman et al
- V–A theory of weak force (1958) by R. Feynman, M. Gell-Mann; …
- Neutrino's negative helicity (1958) by M. Goldhaber et al
- All neutrinos were discovered in US

♦ A consensus on neutrinos for most physicists at that time: massless & left-handed Weyl fermions.

It is a reasonable assumption

- ♦ **T.D. Lee and C.N. Yang, L. Landau, and A. Salam all conjectured neutrinos to be massless in 1957.**



1928 **1929** **1937**

- ♦ **But B. Pontecorvo, a key member of the Fermi school, believed that neutrinos should be massive.**

"Mesonium and Anti-mesonium" in *Sov. Phys. JETP 6 (1957) 429*
If the two-component neutrino theory turned out to be incorrect and if the conservation law of neutrino charge didn't apply, then neutrino—antineutrino *transitions* would in principle be possible to take place in vacuum.

Weinberg's razor

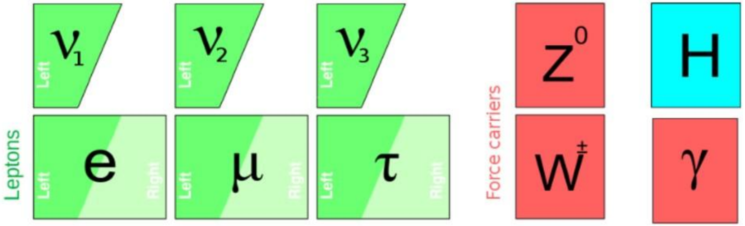
VOLUME 19, NUMBER 21 PHYSICAL REVIEW LETTERS 20 NOVEMBER 1967

Citations ~ 15000

A MODEL OF LEPTONS*

Steven Weinberg†
Laboratory for Nuclear Science and Physics Department,
Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received 17 October 1967)



- It is a *renormalizable* $SU(2)_L \times U(1)_Y$ gauge theory.
- Its particle content is so *economical* that there is no way to make neutrinos massive.

- A Dirac neutrino mass term impossible.
- A Majorana neutrino mass term impossible.
- An effective (non-renormalizable) neutrino mass term impossible.

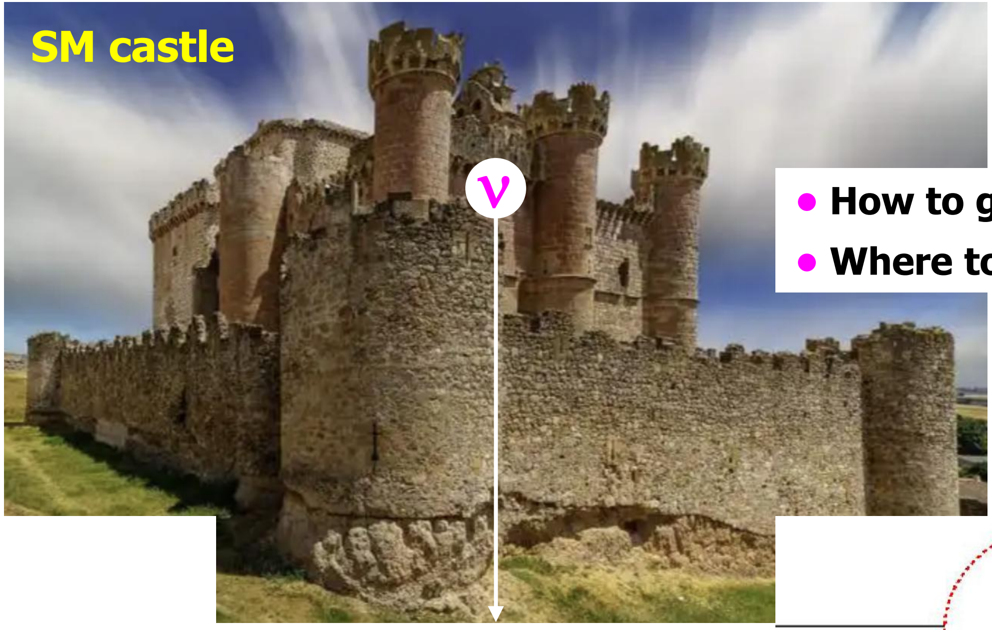
But neutrinos turned out to be massive

	EX discovery or TH breakthrough	Main contributors (★ Nobel laureates)
1937	Majorana fermion	E. Majorana
1957	neutrino-antineutrino transitions	B. Pontecorvo
1962	neutrino flavor mixing	Z. Maki, M. Nakagawa, S. Sakata
1967	formulation of neutrino oscillations	B. Pontecorvo
1968	solar neutrino deficit	R. Davis ★, et al. Weinberg's opinion?
1974	SU(5) GUT + proton decays	H. Georgi, S.L. Glashow
1977	Minkowski (seesaw) mechanism	P. Minkowski
1986	leptogenesis	M. Fukugita, T. Yanagida
1987	supernova neutrinos	M. Koshiba ★, et al.
1998	atmospheric neutrino oscillations	T. Kajita ★, et al. (Super-K)
2001	solar neutrino oscillations	A.B. McDonald ★, et al. (SNO)
2002	reactor long-baseline oscillations	KamLAND
2002	accelerator disappearance oscillations	K2K
2011	accelerator appearance oscillations	T2K
2012	reactor short-baseline oscillations	Daya Bay and RENO

new physics

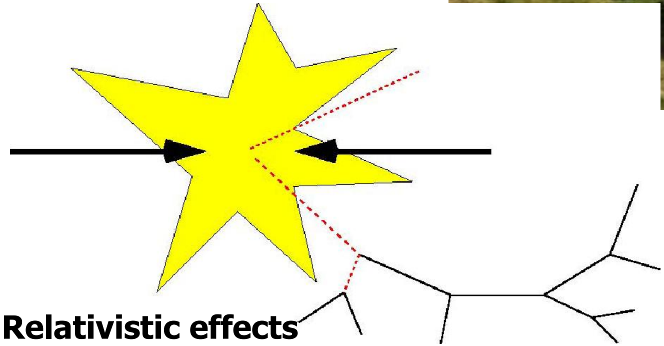
A brief summary

♦ The **weakest** part of the SM leaves us a small window to **see** outside —— a new physics **landscape**.

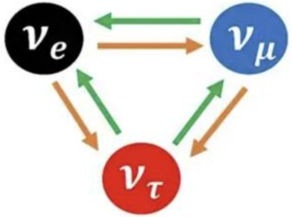


- How to go out of the castle?
- Where to go and what to do

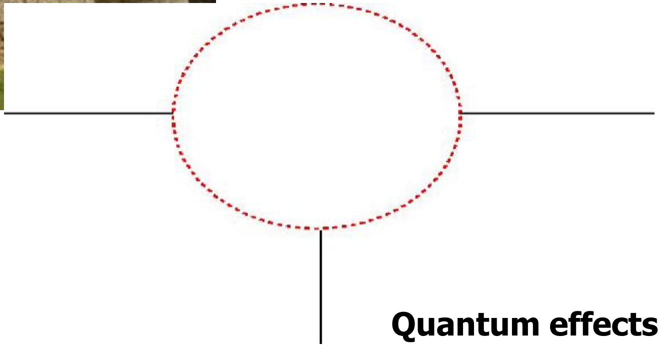
Energy frontier



Relativistic effects



Intensity frontier



Quantum effects

OUTLINE

- Why neutrinos and why massless?
- Why Majorana and why not Dirac?
- Possible ways to test the seesaw?
- There is a flavor symmetry behind
- Inconclusive remarks

Way 1: from Weinberg's SMEFT

♦ It was **S. Weinberg** who invented the ***SMEFT*** in **1979**, as a low-energy effective way to go beyond the SM, in particular to generate tiny **Majorana** neutrino masses.

The standard electroweak theory

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm gauge} + \mathcal{L}_{\rm Higgs} + \mathcal{L}_{\rm fermion} + \mathcal{L}_{\rm Yukawa}$$

(in which neutrinos are massless)

$$\mathcal{L}_{\rm G} = -\frac{1}{4}\left(W^{i\mu\nu}W^{i}_{\mu\nu} + B^{\mu\nu}B_{\mu\nu}\right)$$

$$\mathcal{L}_{\rm H} = (D^{\mu}H)^{\dagger}(D_{\mu}H) - \mu^2 H^{\dagger}H - \lambda(H^{\dagger}H)^2$$

$$\mathcal{L}_{\rm F} = \overline{Q_{\rm L}}\,{\rm i}\not{D}\,Q_{\rm L} + \overline{\ell_{\rm L}}\,{\rm i}\not{D}\,\ell_{\rm L} + \overline{U_{\rm R}}\,{\rm i}\not{\partial}'\,U_{\rm R} + \overline{D_{\rm R}}\,{\rm i}\not{\partial}'\,D_{\rm R} + \overline{E_{\rm R}}\,{\rm i}\not{\partial}'\,E_{\rm R}$$

$$\mathcal{L}_{\rm Y} = -\overline{Q_{\rm L}}\,Y_{\rm u}\tilde{H}U_{\rm R} - \overline{Q_{\rm L}}\,Y_{\rm d}HD_{\rm R} - \overline{\ell_{\rm L}}\,Y_{l}HE_{\rm R} + {\rm h.c.}$$

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_{\forall i, n\geq 5} \frac{C_i \mathcal{O}_i^{(n)}}{\Lambda^{n-4}}$$

Higher (mass) dimension operators suppressed by NP scale

I must say that I am not a model builder!

♦ **Weinberg**: My style is usually not to propose **specific models** that will lead to specific experimental predictions, but rather to interpret ***in a broad way*** what is going on and make very **general remarks**, like with the development of the viewpoint associated with **effective field theory**. (**CERN Courier 2021**)

The unique Weinberg operator

♦ **The SMEFT approach leads us to the first and unique non-renormalizable dimension-five operator, written out by S. Weinberg in 1979:**

"unique" d=5 operator

$$\frac{1}{2} C_{\alpha\beta} \frac{\overline{\ell_{\alpha L}} \widetilde{H} \widetilde{H}^T \ell^c_{\beta L}}{\Lambda}$$

- **tiny neutrino masses**
- **the Majorana nature**

(B—L) violation

spontaneous symmetry breaking

$$-\mathcal{L}_{\rm mass} = \frac{1}{2} \overline{\nu_L} M_\nu \nu^c_L + {\rm h.c.}$$

Naturally tiny: $\mathcal{O}(0.1)$ eV

$$(M_\nu)_{\alpha\beta} = C_{\alpha\beta} \frac{\langle H \rangle^2}{\Lambda}$$

$$C_{\alpha\beta} \sim \mathcal{O}(1), \ \langle H \rangle \sim 10^2 \ {\rm GeV}$$
$$\Lambda \sim 10^{14} \ {\rm GeV}$$

Way 2: the Minkowski mechanism

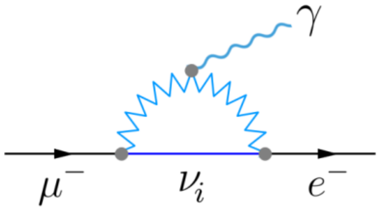
♦ **Pure left handedness and vanishing masses of the active neutrinos, together with lepton number and flavor conservation, are seemingly true in the SM.**

♦ **But they are most likely to belong to the limiting case of a most natural/economical extension of the SM with three right-handed neutrino fields and their self interactions.**

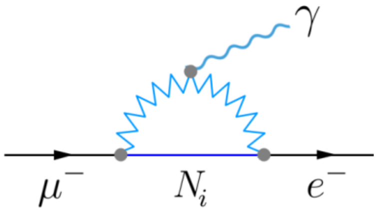
♦ **It was Peter Minkowski who first proposed a nice mechanism in 1977, but named as "seesaw" by others, to naturally arrive at tiny neutrino masses and feeble cLFV effects.**

The title of Peter's paper : $\mu \to e + \gamma$ at a rate of one out of 10^9 muon decays?

P. Minowski, Phys. Lett. B 67 (1977) 421



light neutrinos



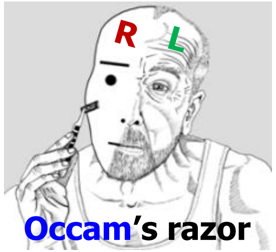
heavy neutrinos

♦ **Amazingly, such a LFV process has offered the most stringent constraints on seesaw parameters.**

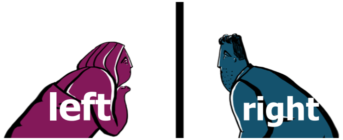
It’s most natural and economical

♦ Neutrinos surely have the *right* to be *right* (-handed) to keep an analogous *left-right symmetry* as charged leptons or quarks.

$$\begin{pmatrix} u_{\rm L} \\ d_{\rm L} \end{pmatrix} \Longleftrightarrow \begin{matrix} u_{\rm R} \\ d_{\rm R} \end{matrix} \qquad \begin{pmatrix} \nu_{e\rm L} \\ e_{\rm L} \end{pmatrix} \Longleftrightarrow \begin{matrix} ? \\ e_{\rm R} \end{matrix}$$

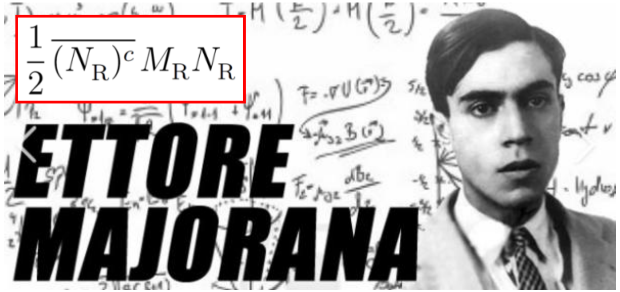


Please note that right-handed fields are NoT the mirror counterparts of left-handed ones.



♦ Then neutrinos are allowed to couple to the SM Higgs doublet —— the Yukawa interactions. Why not?

♦ But the gender of neutrinos (neutral) makes it very fair to add a Majorana mass term with *N* and *N*^{c}, which is fully *harmless* to all the fundamental symmetries of the SM.



♦ Then we are led to the Minkowski mechanism, which works even before SSB (ZZX, 2023):

$$\mathcal{L}_{\nu\rm SM} = \mathcal{L}_{\rm SM} + \overline{N_{\rm R}}\,{\rm i}\gamma_\mu\partial^\mu N_{\rm R} - \left[\overline{\ell_{\rm L}}\,Y_\nu \widetilde{H} N_{\rm R} + \frac{1}{2}\overline{(N_{\rm R})^c}\,M_{\rm R} N_{\rm R} + {\rm h.c.}\right]$$

(B—L) violation

$$= \mathcal{L}_{\rm SM} + \overline{N_{\rm R}}\,{\rm i}\gamma_\mu\partial^\mu N_{\rm R} - \left[\frac{1}{2}\overline{\left[\nu_{\rm L} \;\; (N_{\rm R})^c\right]}\begin{pmatrix} \mathbf{0} & Y_\nu \phi^{0*} \\ Y_\nu^T \phi^{0*} & M_{\rm R}\end{pmatrix}\begin{bmatrix} (\nu_{\rm L})^c \\ N_{\rm R}\end{bmatrix} - \overline{l_{\rm L}}\,Y_\nu N_{\rm R}\phi^- + {\rm h.c.}\right]$$

The exact seesaw formula

♦ A basis transformation to obtain **Majorana** neutrino masses and flavor mixing before or after **SSB**.

$$\mathbb{U}^\dagger \begin{pmatrix} \mathbf{0} & Y_\nu \phi^{0*} \\ Y_\nu^T \phi^{0*} & M_{\rm R} \end{pmatrix} \mathbb{U}^* = \begin{pmatrix} D_\nu & \mathbf{0} \\ \mathbf{0} & D_N \end{pmatrix}$$

The three-block decomposition: **ZZX**, **1110.0083**

$$\mathbb{U} = \begin{pmatrix} I & \mathbf{0} \\ \mathbf{0} & U_0' \end{pmatrix} \begin{pmatrix} A & R \\ S & B \end{pmatrix} \begin{pmatrix} U_0 & \mathbf{0} \\ \mathbf{0} & I \end{pmatrix}$$

sterile (unitary) — **Yukawa (interplay)** — **active (unitary)**

$$(O_{56}O_{46}O_{45})\,[O_{36}O_{26}O_{16}O_{35}O_{25}O_{15}O_{34}O_{24}O_{14}]\,(O_{23}O_{13}O_{12})$$

working masses:

$$\begin{cases} D_\nu \equiv {\rm Diag}\{m_1, m_2, m_3\} \\ D_N \equiv {\rm Diag}\{M_4, M_5, M_6\} \end{cases}$$

♦ **Weak charged-current interactions of leptons in the seesaw mechanism:**

$$-\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(e \quad \mu \quad \tau)_{\rm L}} \gamma^\mu \left[U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} + R \begin{pmatrix} N_4 \\ N_5 \\ N_6 \end{pmatrix}_{\rm L} \right] W_\mu^- + {\rm h.c.}$$

oscillations, LNV, LFV ← light — **heavy → collider, LNV, LFV**

$U = AU_0$: **the PMNS matrix**

R : **an analogue for heavy**

$$U D_\nu U^T = ({\rm i}R)\, D_N\, ({\rm i}R)^T$$

$$UU^\dagger + RR^\dagger = I$$ **(unitarity relation)**

- **The PMNS matrix U is not exactly unitary in the seesaw scenario**
- **But non-unitarity of U is constrained to be very small**

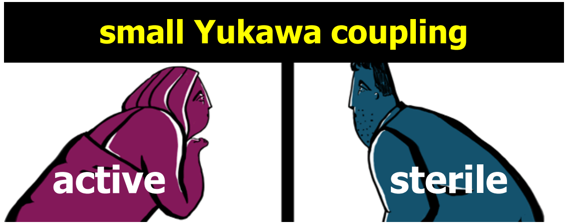
How to make masses tiny and flavor mixing big?

- In the canonical seesaw framework, it is technically natural to make ν-masses as tiny as possible:

$$U_0 D_\nu U_0^T = \left(\mathrm{i}A^{-1}R\right) D_N \left(\mathrm{i}A^{-1}R\right)^T \longrightarrow m_1 m_2 m_3 = M_4 M_5 M_6 \left[\det\left(\mathrm{i}A^{-1}R\right)\right]^2$$

determinants of the two sides

tiny = huge × suppressor



$$A^{-1}R = \begin{pmatrix} \hat{s}^*_{14} & \hat{s}^*_{15} & \hat{s}^*_{16} \\ \hat{s}^*_{24} & \hat{s}^*_{25} & \hat{s}^*_{26} \\ \hat{s}^*_{34} & \hat{s}^*_{35} & \hat{s}^*_{36} \end{pmatrix} + \mathcal{O}\left(s^3_{ij}\right)$$

- But how can we qualitatively see that large flavor mixing angles originate from the sterile sector?

Large flavor mixing of three active neutrinos is an *emergent* effect

The approximate mu-tau reflection symmetry may exist in the sterile sector

because they are highly structure-dependent

A Euler-like parametrization

♦ **The 1st full Euler-like parametrization of $U = AU_0$ and R is useful for calculating flavor structures.**

$$U_0 = \begin{pmatrix} c_{12}c_{13} & \hat{s}^*_{12}c_{13} & \hat{s}^*_{13} \\ -\hat{s}_{12}c_{23} - c_{12}\hat{s}_{13}\hat{s}^*_{23} & c_{12}c_{23} - \hat{s}^*_{12}\hat{s}_{13}\hat{s}^*_{23} & c_{13}\hat{s}^*_{23} \\ \hat{s}_{12}\hat{s}_{23} - c_{12}\hat{s}_{13}c_{23} & -c_{12}\hat{s}_{23} - \hat{s}^*_{12}\hat{s}_{13}c_{23} & c_{13}c_{23} \end{pmatrix}$$

derivational parameters!

$$c_{ij} \equiv \cos\theta_{ij}$$

$$\hat{s}_{ij} \equiv e^{i\delta_{ij}} \sin\theta_{ij} \ (\text{for } 1 \le i < j \le 6)$$

$$A = \begin{pmatrix} c_{14}c_{15}c_{16} & 0 & 0 \\ -c_{14}c_{15}\hat{s}_{16}\hat{s}^*_{26} - c_{14}\hat{s}_{15}\hat{s}^*_{25}c_{26} - \hat{s}_{14}\hat{s}^*_{24}c_{25}c_{26} & c_{24}c_{25}c_{26} & 0 \\ -c_{14}c_{15}\hat{s}_{16}c_{26}\hat{s}^*_{36} + c_{14}\hat{s}_{15}\hat{s}^*_{25}\hat{s}_{26}\hat{s}^*_{36} - c_{14}\hat{s}_{15}c_{25}\hat{s}^*_{35}c_{36} + \hat{s}_{14}\hat{s}^*_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} + \hat{s}_{14}\hat{s}^*_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} - \hat{s}_{14}c_{24}\hat{s}^*_{34}c_{35}c_{36} & -c_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} - c_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} - \hat{s}_{24}\hat{s}^*_{34}c_{35}c_{36} & c_{34}c_{35}c_{36} \end{pmatrix}$$

ZZX 0709.2220/1110.0083

$$R = \begin{pmatrix} \hat{s}^*_{14}c_{15}c_{16} & \hat{s}^*_{15}c_{16} & \hat{s}^*_{16} \\ -\hat{s}^*_{14}c_{15}\hat{s}_{16}\hat{s}^*_{26} - \hat{s}^*_{14}\hat{s}_{15}\hat{s}^*_{25}c_{26} + c_{14}\hat{s}^*_{24}c_{25}c_{26} & -\hat{s}^*_{15}\hat{s}_{16}\hat{s}^*_{26} + c_{15}\hat{s}^*_{25}c_{26} & c_{16}\hat{s}^*_{26} \\ -\hat{s}^*_{14}c_{15}\hat{s}_{16}c_{26}\hat{s}^*_{36} + \hat{s}^*_{14}\hat{s}_{15}\hat{s}^*_{25}\hat{s}_{26}\hat{s}^*_{36} - \hat{s}^*_{14}\hat{s}_{15}c_{25}\hat{s}^*_{35}c_{36} - c_{14}\hat{s}^*_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} - c_{14}\hat{s}^*_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} + c_{14}c_{24}\hat{s}^*_{34}c_{35}c_{36} & -\hat{s}^*_{15}\hat{s}_{16}c_{26}\hat{s}^*_{36} - c_{15}\hat{s}^*_{25}\hat{s}_{26}\hat{s}^*_{36} + c_{15}c_{25}\hat{s}^*_{35}c_{36} & c_{16}c_{26}\hat{s}^*_{36} \end{pmatrix}$$

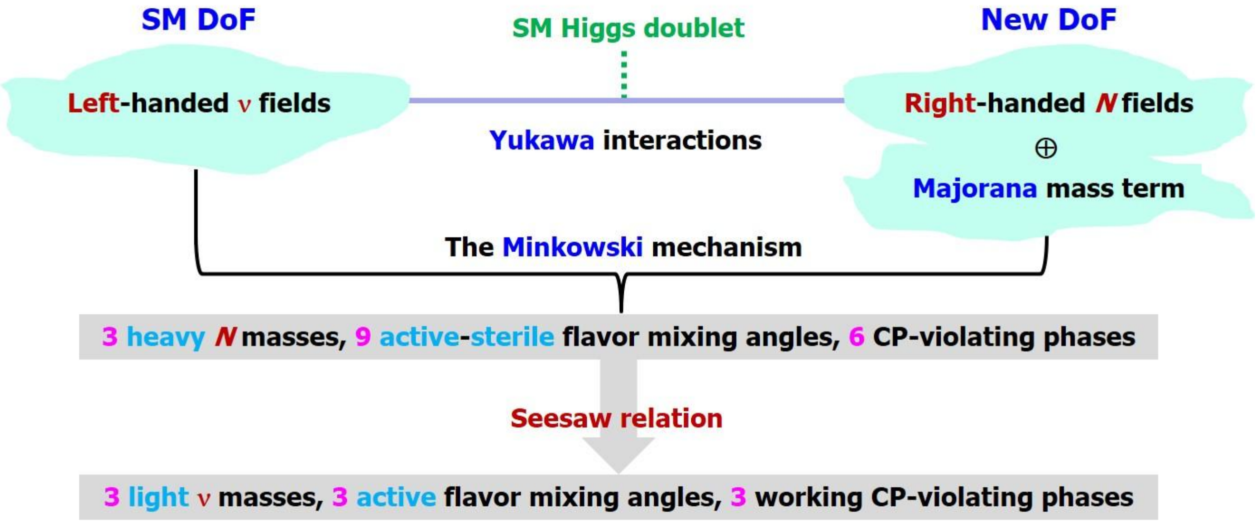
The latest stringent *bounds* on possible PMNS nonunitarity.
M. Blennow et al. 2023

$$\begin{cases} \theta_{1j} < 2.92^\circ \\ \theta_{2j} < 0.27^\circ \\ \theta_{3j} < 2.56^\circ \\ [j = 4, 5, 6] \end{cases}$$

ZZX, J. Zhu, 2412.17698

How many parameters in total?

- ♦ The *canonical* seesaw mechanism contains 18 original parameters, giving rise to 9 effective ones.



- **Original:** $M_j \ (\text{for } j = 4, 5, 6) \ ; \quad \theta_{ij} \ (\text{for } i = 1, 2, 3; j = 4, 5, 6) \ ; \quad \alpha_i \equiv \delta_{i4} - \delta_{i5} \ , \ \beta_i \equiv \delta_{i5} - \delta_{i6} \ (\text{for } i = 1, 2, 3)$
- **Derivational:** $m_i \ (\text{for } i = 1, 2, 3) \ ; \quad \theta_{ij} \ (\text{for } j > i = 1, 2, 3) \ ; \quad \delta_{ij} \ (\text{for } j > i = 1, 2, 3)$

- ♦ A mixture of the two sets of parameters (like the Casas-Ibarra parametrization) might be confusing:

$$Y_\nu \simeq \frac{\mathrm{i}}{\langle H \rangle} U_0 \sqrt{D_\nu} O \sqrt{D_N}$$

O : arbitrary orthogonal matrix.

From seesaw to SMEFT

♦ **Integrating out the heavy degrees of freedom —— right-handed neutrino fields, one may arrive at the seesaw EFT matching with the SMEFT (see A. Broncano, M.B. Gavela, E. Jenkins 2003; A. Abada, C. Biggio, F. Bonnet, M.B. Gavela, T. Hambye 2007; D. Zhang, S. Zhou 2021; ...).**

$$\mathcal{L}_{\nu\mathrm{SM}} = \mathcal{L}_{\mathrm{SM}} + \left[\frac{1}{2} C^{(5)}_{\alpha\beta} \frac{\mathcal{O}^{(5)}_{\alpha\beta}}{\Lambda} + \mathrm{h.c.}\right] + C^{(6)}_{\alpha\beta} \frac{\mathcal{O}^{(6)}_{\alpha\beta}}{\Lambda^2} + \cdots$$

At the tree level,

$$\mathcal{O}^{(5)}_{\alpha\beta} = \left(\overline{\ell_{\alpha\mathrm{L}}} \widetilde{H}\right)\left(\widetilde{H}^T \ell^c_{\beta\mathrm{L}}\right)$$

The Weinberg operator: (B–L) violating

$$\mathcal{O}^{(6)}_{\alpha\beta} = \left(\overline{\ell_{\alpha\mathrm{L}}} \widetilde{H}\right) \mathrm{i}\gamma_\mu \partial^\mu \left(\widetilde{H}^\dagger \ell_{\beta\mathrm{L}}\right)$$

The PMNS non-unitarity: (B–L) conserving

$$\frac{C^{(5)}_{\alpha\beta}}{\Lambda} = \left(Y_\nu M_{\mathrm{R}}^{-1} Y_\nu^T\right)_{\alpha\beta}$$

$$\frac{C^{(6)}_{\alpha\beta}}{\Lambda^2} = \left(Y_\nu |M_{\mathrm{R}}|^{-2} Y_\nu^\dagger\right)_{\alpha\beta}$$

$$U = \left[I - \frac{1}{2}\langle H\rangle^2 \left(Y_\nu |M_{\mathrm{R}}|^{-2} Y_\nu^\dagger\right)\right] U_0^{(5)} = \left[I - \frac{1}{2} R R^\dagger\right] U_0^{(5)}$$

PMNS non-unitarity

♦ **So there are two kinds of parametrizations of the PMNS matrix in the seesaw framework:**

- **Motivated by the seesaw EFT (A. Broncano, M.B. Gavela, E. Jenkins 2003):**
  PMNS matrix = Hermitian matrix × unitary matrix (from d=5 operator)

$$U \simeq \left[I - \frac{1}{2} \begin{pmatrix} 2a_{11} & a_{21}^* & a_{31}^* \\ a_{21} & 2a_{22} & a_{32}^* \\ a_{31} & a_{32} & 2a_{33} \end{pmatrix} \right] U_0^{(5)}$$

- **Motivated by a full seesaw parametrization (ZZX, 0709.2220/1110.0083):**
  PMNS matrix = Lower triangular matrix × unitary matrix ($U = AU_0$)

$$U \simeq \left[I - \begin{pmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \right] U_0$$

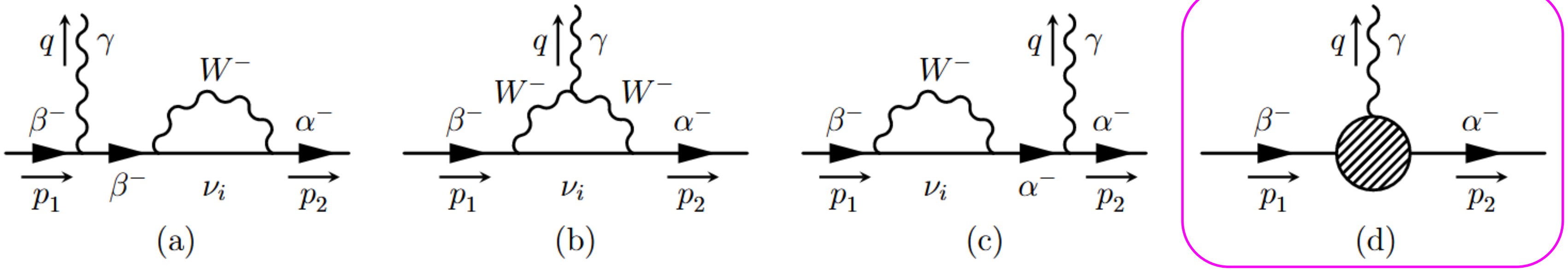
$$a_{ii} \equiv \left(s_{i4}^2 + s_{i5}^2 + s_{i6}^2\right)/2 \text{ and } a_{ij} \equiv \hat{s}_{i4}^* \hat{s}_{j4} + \hat{s}_{i5}^* \hat{s}_{j5} + \hat{s}_{i6}^* \hat{s}_{j6}$$

♦ **A detailed analysis of currently available electroweak and flavor precision data leads to stringent constraints on the PMNS non-unitarity (M. Blennow et al. 2023).**

Remember Weinberg's 2nd law

♦ One may check whether there is a consistency between the full seesaw and its EFT by calculating the radiative decays of charged leptons (ZZX, D. Zhang, 2009.09717) :

- Discrepancy: the full seesaw vs its minimal unitarity violation EFT (S. Antusch et al 2006, 2014)
- Consistency: the full seesaw vs its EFT with complete one-loop matching (D. Zhang, S. Zhou 2021)



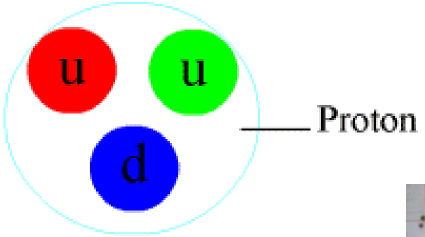
where diagram (d) is generated by the dim-6 operator at the one-loop level and is crucial for the seesaw EFT to correctly calculate the radiative decays of charged leptons.

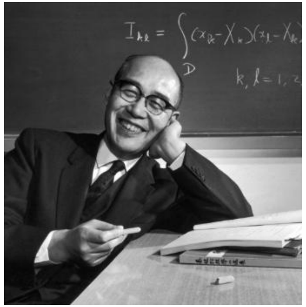
♦ Steven Weinberg's 2nd Law of Progress in Theoretical Physics (1983):

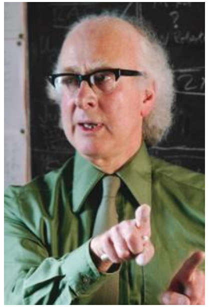
Don't trust arguments based on the lowest order of perturbation theory

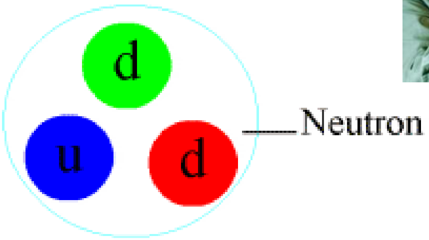
The key is Yukawa interactions

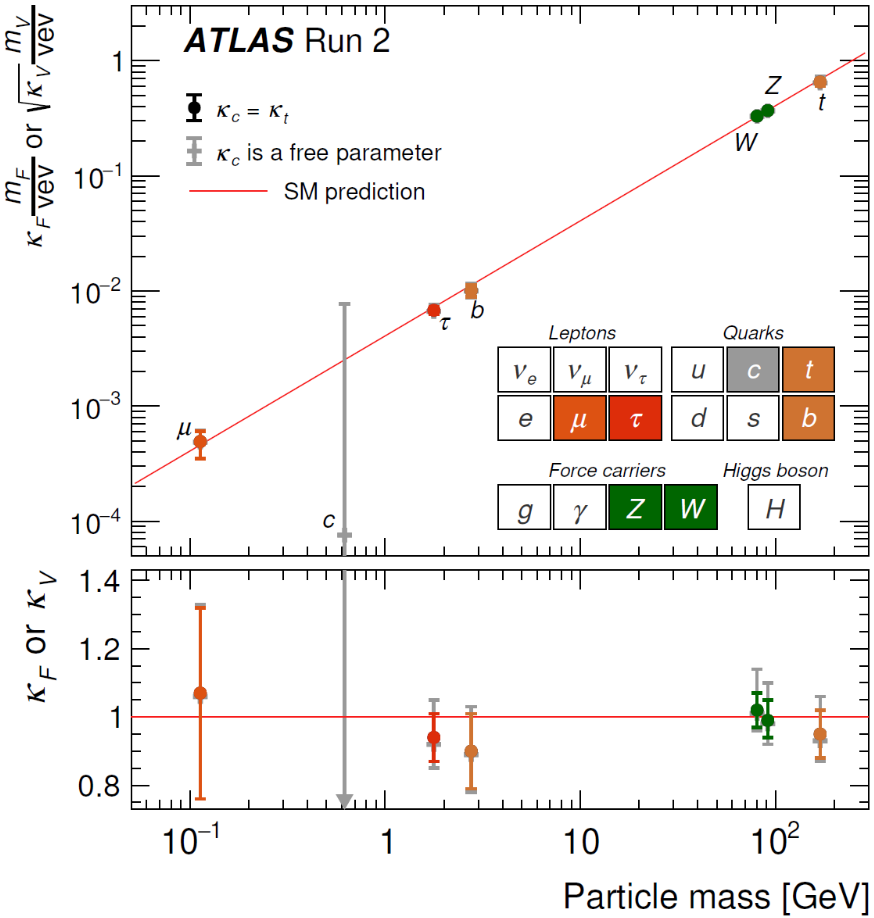
- A **Yukawa interaction** between 2 nucleons (**1935**):











In the SM a Yukawa coupling measures the strength of a fundamental fermion interacting with the Higgs field, from which it gets its finite mass.

How about another seesaw?

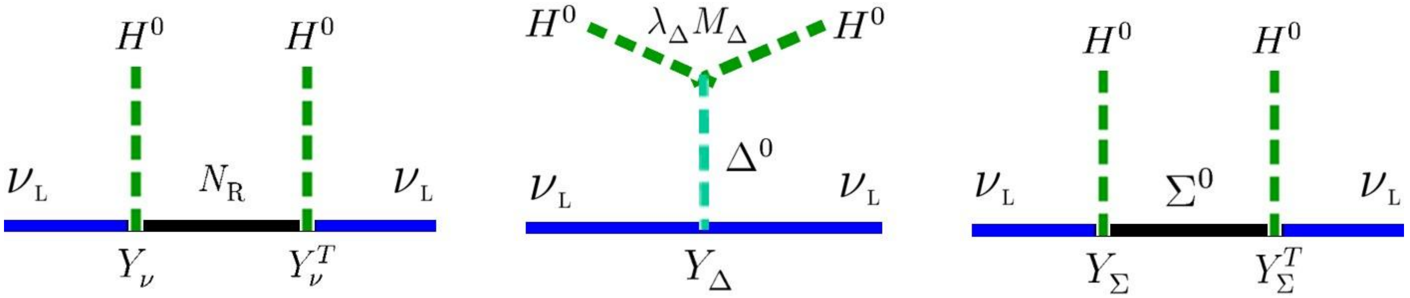
♦ **In extending the boundary of the particle content of the SM to generate small neutrino masses at the tree level, one has also considered the following two typical seesaw mechanisms:**

- **Type-1 seesaw: SM + 3 neutrino singlets (P. Minkowski 1977; …)**
- **Type-2 seesaw: SM + 1 scalar triplet (W. Konetschny, W. Kummer 1977; …)**
- **Type-3 seesaw: SM + 3 fermion triplets (R. Foot, H. Lew, X.G. He, G.C. Joshi 1989)**

common features: Yukawa interactions and (B–L) violation

♦ **After integrating out *heavy degrees of freedom*, we are led to the same d=5 *Weinberg operator*:**

$$\frac{1}{2} C_{\alpha\beta} \frac{\overline{\ell_{\alpha \rm L}} \widetilde{H} \widetilde{H}^T \ell^c_{\beta \rm L}}{\Lambda}$$



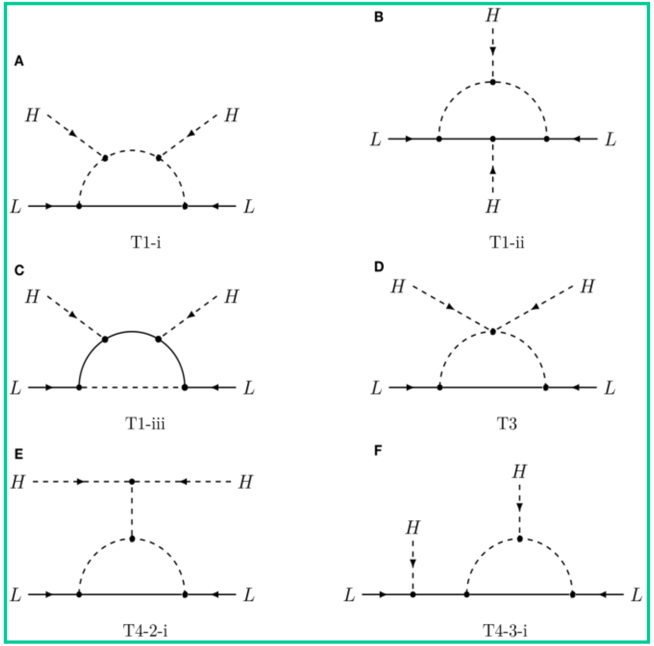
♦ **Given their respective costs and gains, we conclude that "type-1" is most natural and economical.**

How about a radiative origin?

- ♦ **Radiative origin** of charged-lepton and neutrino masses (**S. Weinberg 1972**, **2020**; **A. Zee 1980** …)
- ♦ **A review by Y. Cai, J.H. Garcia, M.A. Schmidt, A. Vicente, R.R. Volkas in Front. in Phys. 5 (2017) 63**
  "from the trees to the forest: a review of radiative neutrino mass models"

Strategy (1): find a way to make the "tree-level" contributions forbidden

Strategy (2): find a way to produce tiny neutrino masses at the loop level



"I am no closer to answering it than I was in the summer of 1972" (2017)

PHYSICAL REVIEW D **101,** 035020 (2020)

Forest = more complicated/expensive?

Models of lepton and quark masses

Steven Weinberg*

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(Received 15 December 2019; accepted 27 January 2020; published 19 February 2020)

A class of models is considered in which the masses only of the third generation of quarks and leptons arise in the tree approximation, while masses for the second and first generations are produced respectively by one-loop and two-loop radiative corrections. So far, for various reasons, these models are not realistic.

Feynman diagram topologies for one-loop radiative neutrino mass generation by the d=5 Weinberg operator, where a dashed line can be scalars or gauge bosons if allowed.

Why not just a Dirac mass?

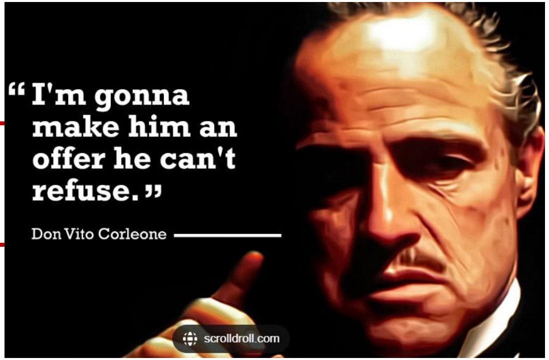
♦ It is cheap to write out a Dirac neutrino mass term, but it is expensive to abandon the associated Majorana mass term —— a buy-one-get-one-free sale, but the "free" one is not really free!

If you like a Dirac mass term: $\overline{\nu_{\alpha L}}(M_D)_{\alpha\beta}\nu_{\beta R}$ No one criticizes your choice.

If you abandon the Majorana term: $\overline{(\nu_{\alpha R})^c}(M_R)_{\alpha\beta}\nu_{\beta R}$ Pls make me an offer I can't refuse.

M. Gell-Mann's totalitarian principle (1956)

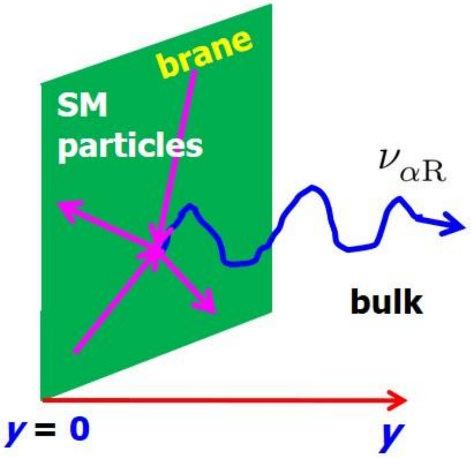
Everything not forbidden is compulsory!



♦ a Dirac neutrino mass model can not naturally be built, unless lepton number symmetry is ad hoc imposed on it with the help of some new physics beyond the SM (R. Volkas, 2409.09992).

A typical Dirac example?

♦ An intriguing way is to invoke extra spatial dimensions beyond the (3+1) structure of spacetime, such that smallness of *Dirac neutrino masses* is attributed to the assumption that the right-handed neutrino fields $\nu_{\alpha \rm R}$ have access to an extra special dimension. After confining the SM particles onto a brane and allowing $\nu_{\alpha \rm R}$ to travel in the bulk, one may obtain the suppressed Yukawa interactions for three active neutrinos located on the brane by *adjusting* the length of the extra dimension over which the wave functions of $\nu_{\alpha \rm R}$ spread out (K. Dienes, E. Dudas, T. Gherghetta 1998).



$$\left[\overline{\ell_{\rm L}} Y_\nu \widetilde{H} \nu_{\rm R}\right]_{y=0} \sim \frac{1}{\sqrt{L}} \left[\overline{\ell_{\rm L}} Y_\nu \widetilde{H} \nu_{\rm R}\right]_{y=L}$$

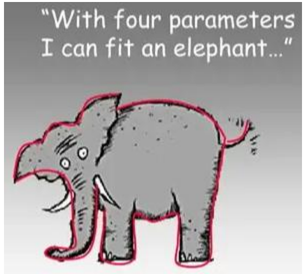
natural suppression of the Yukawa couplings

♦ Note, however, that a Majorana mass term is in general allowed in such a model to make *seesaw* viable. So massive neutrinos are more likely to be the Majorana particles no matter where they are.

Remember Weinberg's 3rd law

♦ Going beyond the SM may either mean going beyond the "3 G" paradigm of fundamental fermions or any other parts of the SM. A lot of attention has been paid to the sterile species of neutrinos, and to other new particles to understand *neutrino mass generation* or some *puzzling anomalies*.

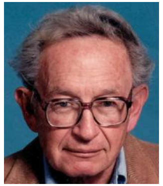
sterile species
- ♦ 3 + 1: light (eV, keV), LSND, warm DM....
- ♦ 3 + 2: heavy (the minimal seesaw)
- ♦ 3 + 3: heavy (the canonical seesaw)
- ♦ 3 + 6: the double or inverse seesaw
- ♦ other possible new particles beyond SM



♦ Steven Weinberg's 3rd Law of Progress in Theoretical Physics (1983):

You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you will be sorry

More = New dynamics!



P. Anderson
More is different (1972)

♦ A good lesson: the history of particle physics tells us that a true new degree of freedom must help solve *at least one* fundamental problem, to make *the theory* more natural, more consistent and more powerful.

A brief summary

♦ Why are theorists often led astray today on the way of searching for new physics beyond the SM?

Bert A.N. Schellekens (2008): The emperor's last clothes? —— guiding or misguiding principles for searching for new physics

- Agreement with observation
- Consistency
- Symmetry
- Simplicity
- Naturalness
- Economy / Occam's razor
- Completeness …

top down
or
bottom up

♦ Introducing the Dirac neutrino masses is by no means simpler and easier than writing a Majorana neutrino mass term, and both of them need new physics.

♦ Majorana neutrinos: new physics, new form of matter, profound and far-reaching implications on particle physics, nuclear physics, cosmology and some other aspects of basic sciences.

OUTLINE

- Why neutrinos and why massless?
- Why Majorana and why not Dirac?
- Possible ways to test the seesaw?
- There is a flavor symmetry behind
- Inconclusive remarks

Retrospect: Murayama's Q & A

♦ **Let's take a brief account of what Hitoshi Murayama was seriously concerned with some time ago.**

PRL **97,** 231801 (2006) PHYSICAL REVIEW LETTERS week ending 8 DECEMBER 2006

How Can We Test the Neutrino Mass Seesaw Mechanism Experimentally?

Matthew R. Buckley and Hitoshi Murayama

Can we prove leptogenesis experimentally? Lay Nam Chang, John Ellis, Belen Gavela, Boris Kayser, and myself got together at Snowmass 2001 and discussed this question. The short answer is unfortunately no. There are additional CP violating phases in the heavy right-handed neutrino sector that cannot be seen by studying the light left-handed neutrinos.[5] For example, even two-generation seesaw mechanism is enough to have CP violation that can potentially produce lepton asymmetry, unlike the minimum of three-generations for CP violation in neutrino oscillation. However, we decided that if we will see (1) electroweak baryogenesis ruled out, (2) lepton-number violation *e.g.* in neutrinoless double beta decay,[6] and (3) CP violation in the neutrino sector *e.g.*, in very long-baseline neutrino oscillation experiment, we will probably believe it based on these "archaeological" evidences.

H. Murayama: "Theory of neutrino masses and mixings", Plenary talk at Lepton-Photon 2001.

Direct way: largely impossible

♦ The not-so-heavy **Majorana** neutrinos could be produced at a super-high energy collider provided they are ***kinetically unforbidden*** and ***dynamically unsuppressed***. Of course, nothing has been found.

$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \overline{\begin{pmatrix} e & \mu & \tau \end{pmatrix}_L} \gamma^\mu \left[U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L + R \begin{pmatrix} N_4 \\ N_5 \\ N_6 \end{pmatrix}_L \right] W^-_\mu + \text{h.c.}$$

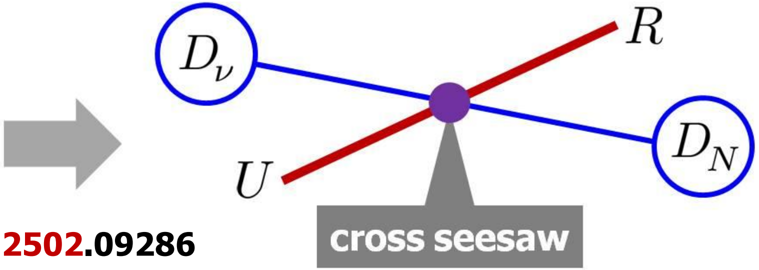
oscillations, LNV, LFV ⟵ light (U)   (R) **heavy ⟶ collider, LNV, LFV**

♦ **Seesaw relation:**

$$U D_\nu U^T = (\text{i}R)\, D_N\, (\text{i}R)^T$$

Exact seesaw:
Cross seesaw:
in mass basis:

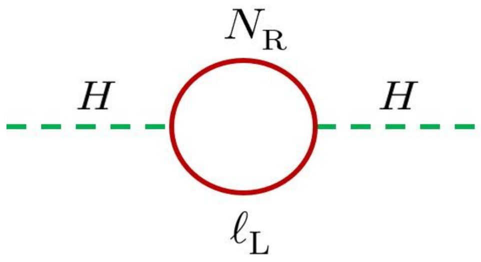
ZZX, 2502.09286



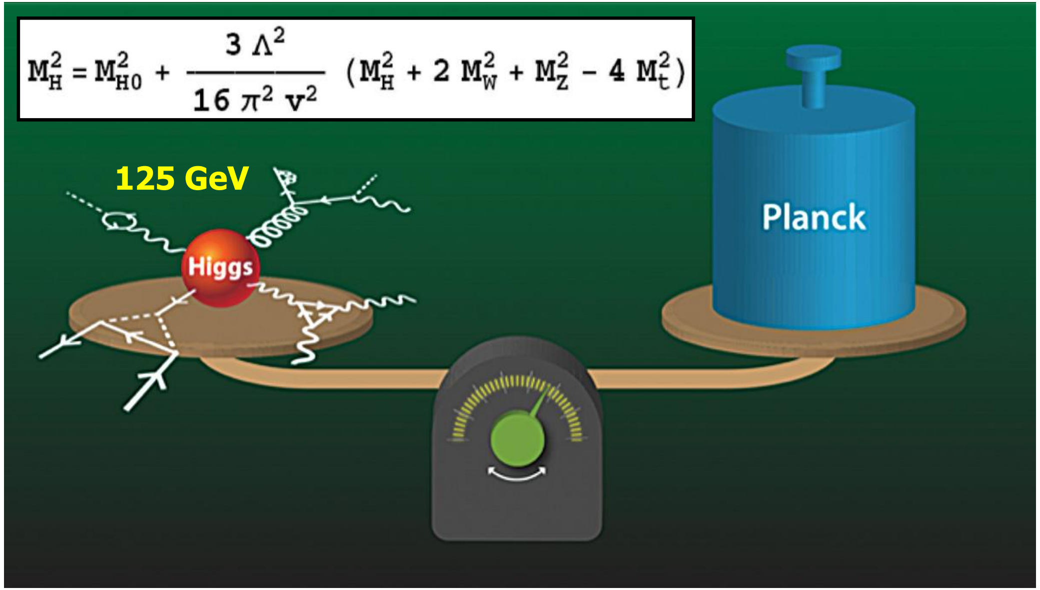
The **larger** the heavy **Majorana** neutrino masses, the **smaller** the **active-sterile** flavor mixing effects, implying that it is **very difficult** to test a **natural seesaw** in an experimentally direct way.

Indirect way 1a: naturalness of the SM

♦ **The seesaw-induced naturalness (fine-tuning) problem: the Higgs mass is sensitive to a quantum correction from heavy degrees of freedom in the seesaw mechanism (F. Vissani 1998; J. Casas et al 2004; A. Abada et al 2007)**



$$\delta m_H^2 = -\frac{y_i^2}{8\pi^2}\left(\Lambda^2 + M_i^2 \ln \frac{M_i^2}{\Lambda^2}\right)$$

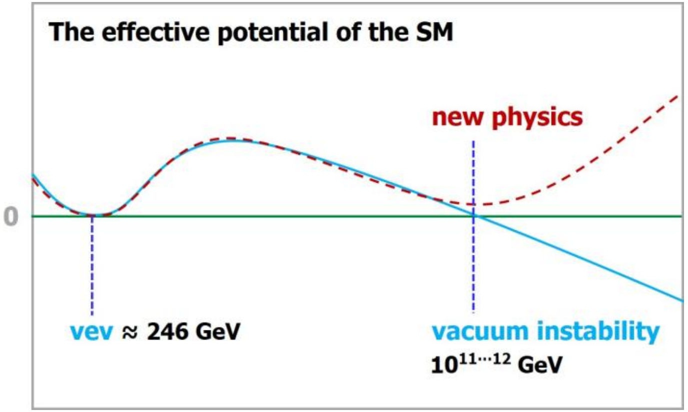


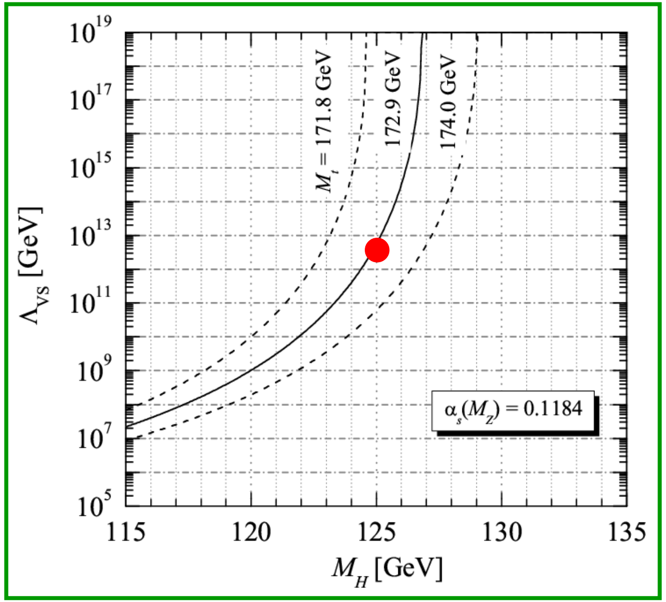
An illustration of *fine-tuning*

$$M_i \sim \left[\frac{(2\pi v)^2|\delta m_H^2|}{m_i}\right]^{1/3} \sim 10^7\text{GeV}\left[\frac{0.2\ \text{eV}}{m_i}\right]^{1/3}\left[\frac{|\delta m_H^2|}{0.1\ \text{TeV}^2}\right]^{1/3}$$

Indirect way 1b: SM vacuum stability

♦ The heavy degrees of freedom in the seesaw mechanism contribute to the renormalization group equations (RGEs) of the SM and thus affect the vacuum stability of the SM (J. Elias-Miro et al 2012, ZZX, H. Zhang, S. Zhou 2012)





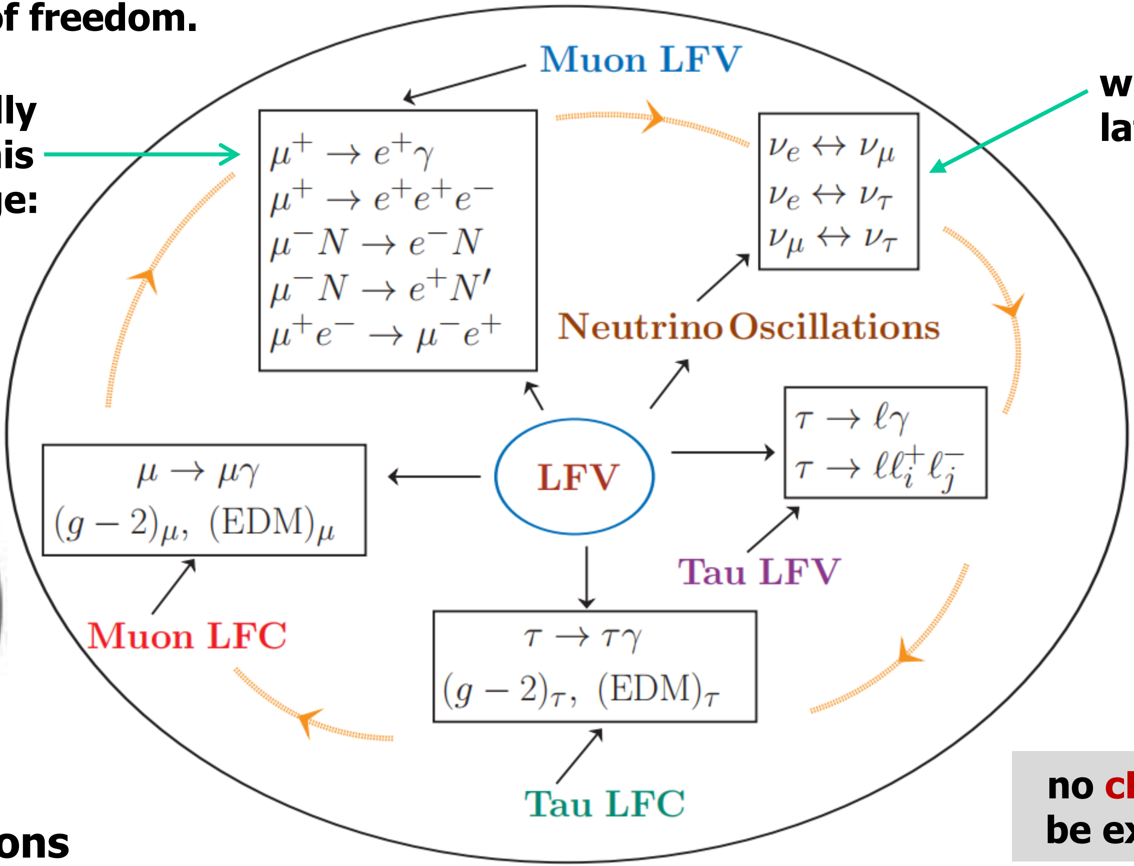
The SM vacuum stability for a light Higgs

♦ A careful calculation of the seesaw-associated *Yukawa-interaction* contribution to the SM RGE is necessary, and this will help constrain the seesaw parameter space.

Indirect way 2: charged LFV

♦ Charged-lepton-flavor-violating processes induced by seesaw are sensitive to parameter space of the heavy degrees of freedom.

seesaw was originally invented to make this guy naturally emerge:



will be discussed later on

J. Albrecht *et al*
Snowmass 2013

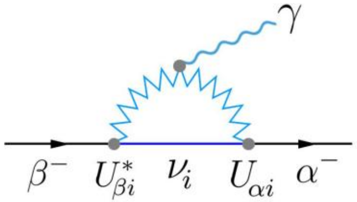
M = neutrinos

D = charged leptons

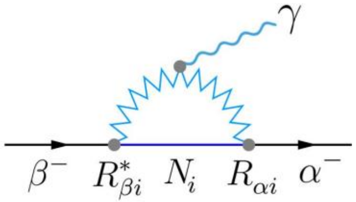
no cLFV effects would be extremely strange!

Radiative cLFV decays

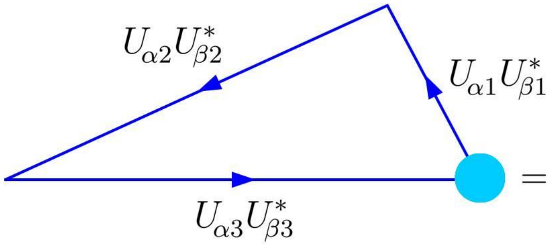
♦ **It can help to constrain unitarity of the 3 × 3 PMNS matrix through the cLFV processes as follows.**



light neutrinos



heavy neutrinos



$$= \sum_{j=4}^{6} \left(R_{\alpha j} R^*_{\beta j}\right)$$

In the full seesaw or its EFT with one-loop matching:

$$\xi_{\alpha\beta} \equiv \frac{\Gamma(\beta^- \to \alpha^- + \gamma)}{\Gamma(\beta^- \to \alpha^- + \overline{\nu}_\alpha + \nu_\beta)} \simeq \frac{3\alpha_{\rm em}}{2\pi} \left| \sum_{i=1}^{3} U_{\alpha i} U^*_{\beta i} \left(-\frac{5}{6} + \frac{1}{4} \cdot \frac{m_i^2}{M_W^2}\right) - \frac{1}{3} \sum_{j=4}^{6} R_{\alpha j} R^*_{\beta j} \right|^2 \simeq \frac{3\alpha_{\rm em}}{8\pi} \left| \sum_{j=4}^{6} R_{\alpha j} R^*_{\beta j} \right|^2$$

which allows us to constrain the unitarity hexagon using current experimental data on the radiative *cLFV* decays:

$$\left| \sum_{i=1}^{3} U_{\alpha i} U^*_{\beta i} \right| = \left| \sum_{j=4}^{6} R_{\alpha j} R^*_{\beta j} \right| \simeq \sqrt{\frac{8\pi \xi_{\alpha\beta}}{3\alpha_{\rm em}}} \simeq 33.88 \sqrt{\xi_{\alpha\beta}}$$

$$\left\{ \begin{array}{l} \left| \sum_{i=1}^{3} U_{e i} U^*_{\mu i} \right| = \left| \sum_{j=4}^{6} R_{e j} R^*_{\mu j} \right| < 2.20 \times 10^{-5} \\ \left| \sum_{i=1}^{3} U_{e i} U^*_{\tau i} \right| = \left| \sum_{j=4}^{6} R_{e j} R^*_{\tau j} \right| < 1.46 \times 10^{-2} \\ \left| \sum_{i=1}^{3} U_{\mu i} U^*_{\tau i} \right| = \left| \sum_{j=4}^{6} R_{\mu j} R^*_{\tau j} \right| < 1.70 \times 10^{-2} \end{array} \right.$$

Indirect way 3: PMNS non-unitarity

♦ **A salient feature of the canonical seesaw mechanism is the tiny *but* nonzero PMNS non-unitarity:**

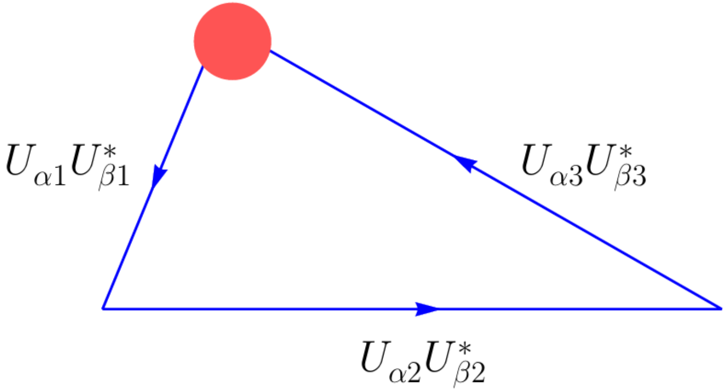
Weak cc-interactions of leptons in the canonical seesaw mechanism:

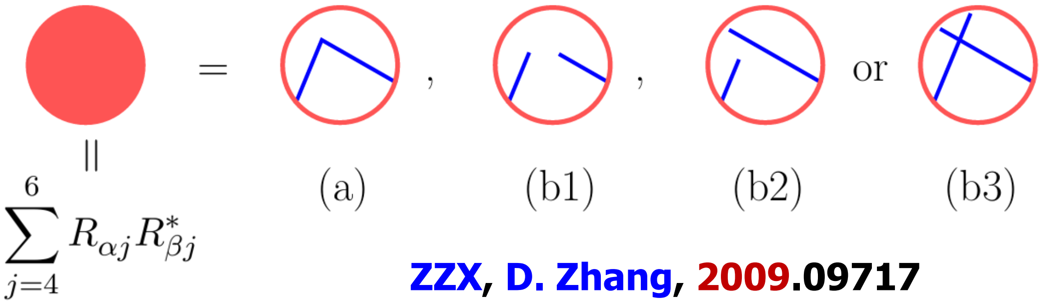
$$-\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{\begin{pmatrix} e & \mu & \tau \end{pmatrix}_{\rm L}} \gamma^\mu \left[U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} + R \begin{pmatrix} N_4 \\ N_5 \\ N_6 \end{pmatrix}_{\rm L} \right] W^-_\mu + {\rm h.c.}$$

$U = AU_0$: the PMNS matrix

R : an analogue for heavy

oscillations, LNV, LFV ← light **heavy → collider, LNV, LFV**





ZZX, D. Zhang, 2009.09717

♦ **To what extent the unitarity hexagon can be treated as an effective triangle? The effective apex has well been constrained by precision electroweak and flavor data and by neutrino oscillation data**

Constraints and implications

♦ **A global fit of all the available data has offered stringent bounds on nonunitarity of the PMNS matrix $U = AU_0$ (M. Blennow et al 2023; ZZX, J.Y. Zhu 2025), allowing us to arrive at the limits on active-sterile flavor mixing angles:**

$$\epsilon \equiv |I - A| < \begin{cases} \text{(1)} \begin{pmatrix} 1.3\times10^{-3} & 0 & 0 \\ 2.4\times10^{-5} & 1.1\times10^{-5} & 0 \\ 1.8\times10^{-3} & 1.1\times10^{-4} & 1.0\times10^{-3} \end{pmatrix} \\ \text{(2)} \begin{pmatrix} 1.4\times10^{-3} & 0 & 0 \\ 2.4\times10^{-5} & 1.0\times10^{-5} & 0 \\ 1.6\times10^{-3} & 3.6\times10^{-5} & 8.1\times10^{-4} \end{pmatrix} \end{cases}$$

There exists a potential problem in this fit.

(1) $m_1 < m_2 < m_3$ **:** $\theta_{1j} < 2.92^\circ \; , \quad \theta_{2j} < 0.27^\circ \; , \quad \theta_{3j} < 2.56^\circ$

(2) $m_3 < m_1 < m_2$ **:** $\theta_{1j} < 3.03^\circ \; , \quad \theta_{2j} < 0.26^\circ \; , \quad \theta_{3j} < 2.31^\circ$

$$A = I - \frac{1}{2}\begin{pmatrix} s^2_{14} + s^2_{15} + s^2_{16} & 0 & 0 \\ 2\hat{s}_{14}\hat{s}^*_{24} + 2\hat{s}_{15}\hat{s}^*_{25} + 2\hat{s}_{16}\hat{s}^*_{26} & s^2_{24} + s^2_{25} + s^2_{26} & 0 \\ 2\hat{s}_{14}\hat{s}^*_{34} + 2\hat{s}_{15}\hat{s}^*_{35} + 2\hat{s}_{16}\hat{s}^*_{36} & 2\hat{s}_{24}\hat{s}^*_{34} + 2\hat{s}_{25}\hat{s}^*_{35} + 2\hat{s}_{26}\hat{s}^*_{36} & s^2_{34} + s^2_{35} + s^2_{36} \end{pmatrix} + \mathcal{O}\left(s^4_{ij}\right)$$

$$R = \begin{pmatrix} \hat{s}^*_{14} & \hat{s}^*_{15} & \hat{s}^*_{16} \\ \hat{s}^*_{24} & \hat{s}^*_{25} & \hat{s}^*_{26} \\ \hat{s}^*_{34} & \hat{s}^*_{35} & \hat{s}^*_{36} \end{pmatrix} + \mathcal{O}\left(s^3_{ij}\right) \longrightarrow U = AU_0 = U_0 + \text{nonunitarity corrections } (\lesssim 10^{-3})$$

Implication: the dimension-six operators are not easily accessible.

The Jarlskog-like invariants

- Of course, one may use the **non-unitary PMNS** matrix ***U*** = ***AU***$_0$ to define the more general ***Jarlskog*** invariants to describe CP violation in neutrino oscillations. But one can show that their **leading terms** are the same, coming from the **unitarity limit**:

$$\mathcal{J}^{ii'}_{\alpha\beta} = \mathcal{J}_\nu + \text{corrections}$$

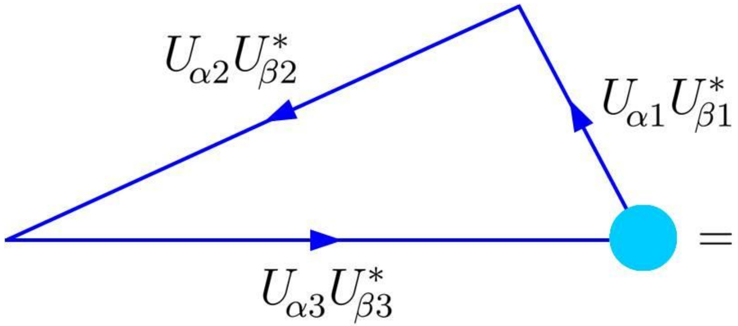
$\leq$ **1%** $\quad$ < **0.01%**

$$\mathcal{J}^{ii'}_{\alpha\beta} \equiv \text{Im}\left(U_{\alpha i} U_{\beta i'} U^*_{\alpha i'} U^*_{\beta i}\right)$$

(α, β) run cyclically over (e, μ, τ)

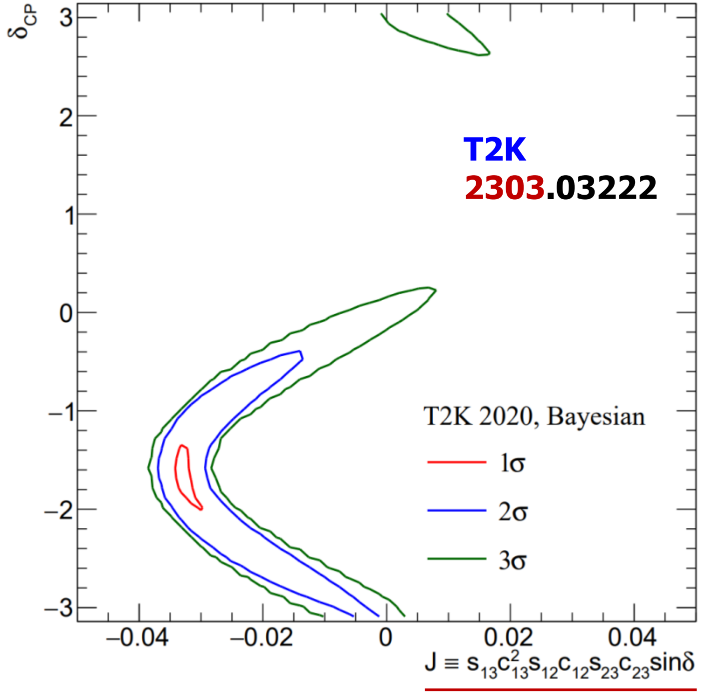
(i, i') run cyclically over $(1, 2, 3)$

- **Yes, absolutely safe, at least by 2045!**



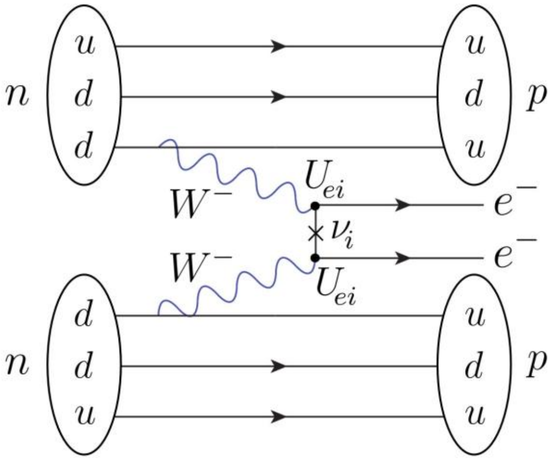
$$= \sum_{j=4}^{6} \left(R_{\alpha j} R^*_{\beta j}\right)$$

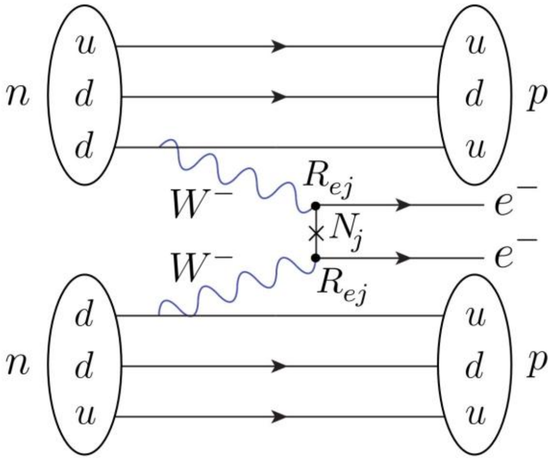
$\alpha = e$
$\beta = \mu$



Indirect way 4: LNV 0ν2β decays

♦ The **seesaw**-induced **Majorana** nature of massive neutrinos allows ***lepton-number-violating 0ν2β*** decays to occur, a unique way to hunt for **Majorana** neutrinos.





Seesaw + Unitarity:

$$\sum_{i=1}^{3} m_i U_{ei}^2 + \sum_{j=4}^{6} M_j R_{ej}^2 = 0$$

$$\sum_{i=1}^{3} |U_{ei}|^2 + \sum_{j=4}^{6} |R_{ej}|^2 = 1$$

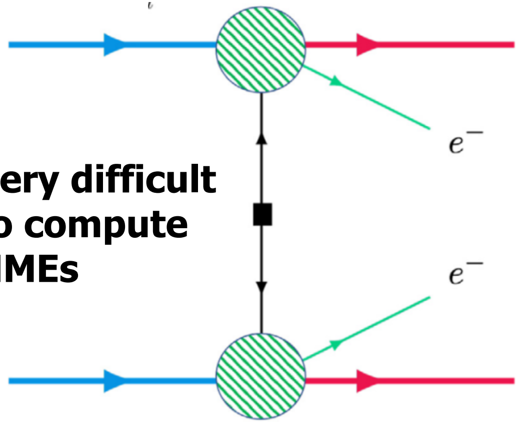
Interplay: propagators + NMEs

Q: Which channel is more fundamental?

A: They are equally fundamental due to Yukawa interactions.

♦ **In most cases the contribution from heavy dof to *0ν2β* are negligible (ZZX, 0907.3014; W. Rodejohann, 0912.3388).**

♦ **Some brave authors have tried to lower the seesaw scales.**

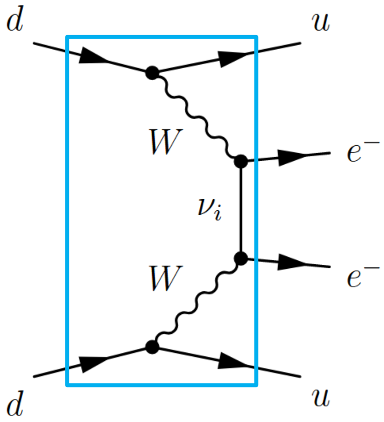


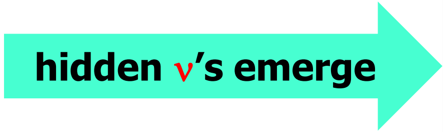
Very difficult to compute NMEs

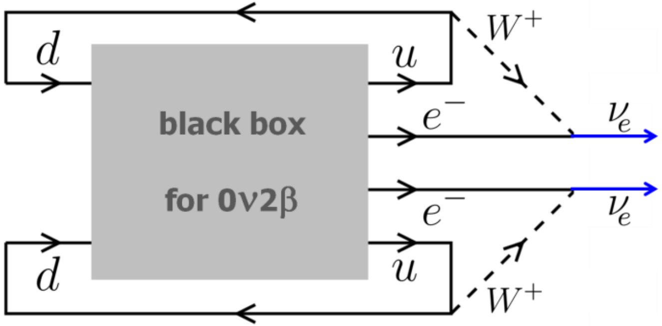
J.M. Yao et al, a review in PPNP 2022

Two theorems about Majorana

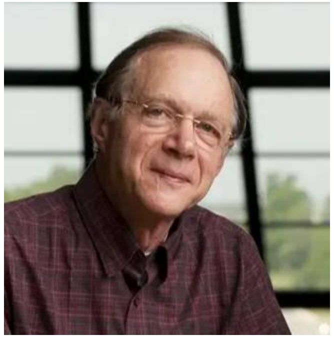
♦ **Joseph Schechter** and **Jose Valle** suggested a theorem in **June 1982**: if a **0ν2β** decay happens, there must exist an effective ***Majorana*** mass term. The reverse is also true.





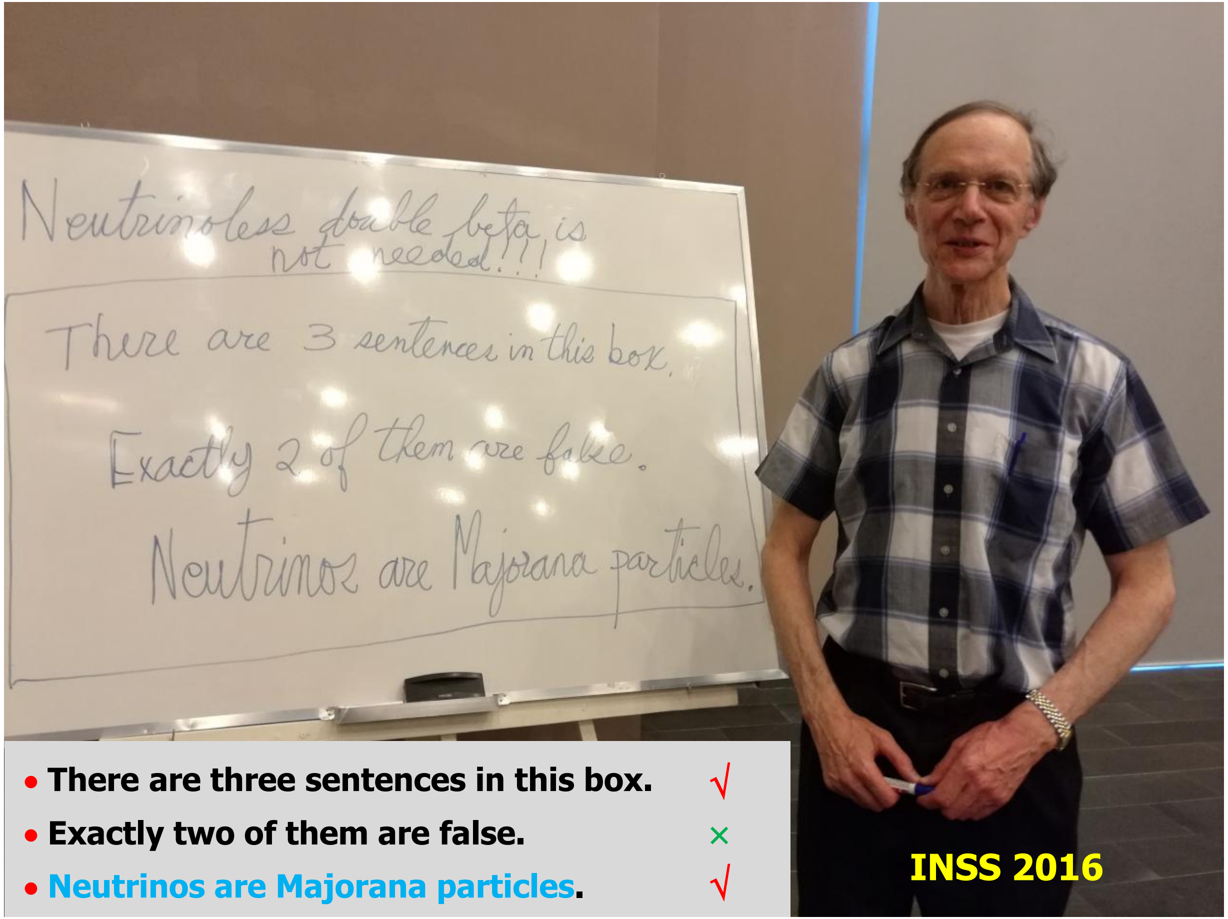


♦ The **Majorana-Dirac confusion theorem** by **Boris Kayser** in **October 1982**: If there are no right-handed currents and the ν-masses are very small compared with the experimental energy scale, then it is **impossible** to tell the difference between **Dirac** and **Majorana** ν's.



♦ **Applicability** of this theorem was clarified by **Choong Sun Kim** (**2022—2025**).

Kayser's proof of Majorana



- There are three sentences in this box. √
- Exactly two of them are false. ×
- Neutrinos are Majorana particles. √

Klapdor's fake 0ν2β signal

Modern Physics Letters A, Vol. 16, No. 37 (2001) 2409–2420

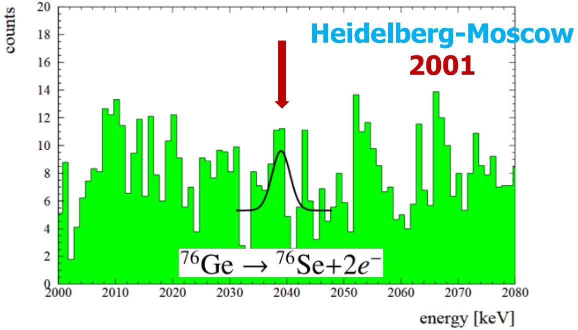
Citations > 700

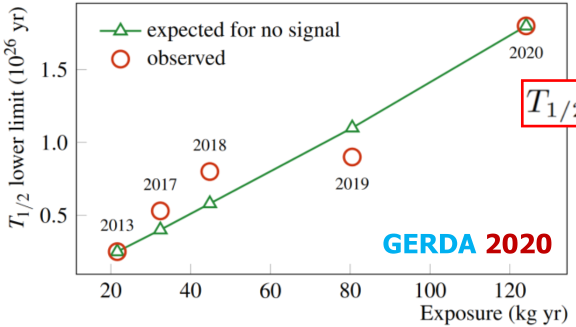
EVIDENCE FOR NEUTRINOLESS DOUBLE BETA DECAY

H. V. KLAPDOR-KLEINGROTHAUS*, A. DIETZ, H. L. HARNEY and I. V. KRIVOSHEINA†

Max-Planck-Institut für Kernphysik, Postfach 103980, D-69029 Heidelberg, Germany

H.V. Klapdor





$T_{1/2} > 1.8 \times 10^{26}$ yr at 90% C.L.

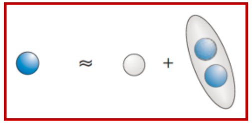
It is not me!

Ettore Majorana

The abstract: First evidence for neutrinoless double beta decay is observed giving first evidence for lepton number violation. The evidence for this decay mode is 97% (2.2σ) with the Bayesian method, and 99.8% c.l. (3.1σ) with the method recommended by the Particle Data Group. The half-life of the process is found with the Bayesian method to be $T^{0\nu}_{1/2} = (0.8 - 18.3) \times 10^{25}$ y (95% c.l.) with a best value of 1.5×10^{25} y. The deduced value of the effective neutrino mass is, with the nuclear matrix elements from [1], $\langle m \rangle = (0.11 - 0.56)$ eV (95% c.l.), with a best value of 0.39 eV. Uncertainties in the nuclear matrix elements

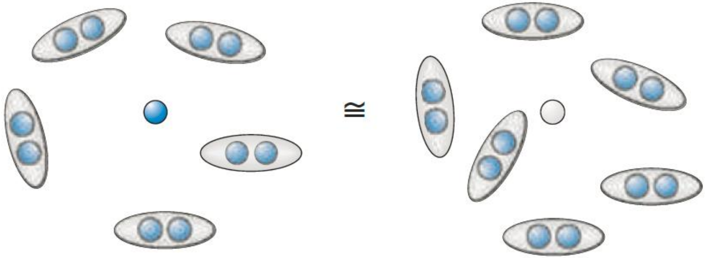
Is Majorana everywhere?

perspective | *Nature Physics* **5**, 614–618 (2009)



Majorana zero mode

Majorana returns



Frank Wilczek

In his short career, Ettore Majorana made several profound contributions. One of them, his concept of 'Majorana fermions' — particles that are their own antiparticle — is finding ever wider relevance in modern physics.

用甜言蜜语哄骗 衍生物、结果

Enrico Fermi had to cajole his friend Ettore Majorana into publishing his big idea: a modification of the Dirac equation that would have profound ramifications for particle physics. Shortly afterwards, in 1938, Majorana mysteriously disappeared, and for 70 years his modified equation remained a rather obscure footnote in 晦涩的 **theoretical physics. Now suddenly, it seems, Majorana's concept is ubiquitous, and his equation is central to recent work not only in neutrino physics, supersymmetry and dark matter, but also on some exotic states of ordinary matter.** 无所不在的、普遍存在的

Zhang's fake evidence

RESEARCH

Majorana ⟶ Devil = Angel

TOPOLOGICAL MATTER

SCIENCE · 21 Jul 2017 · Vol 357, Issue 6348 · pp. 294-299 · DOI: 10.1126/science.aag2792

Chiral Majorana fermion modes in a quantum anomalous Hall insulator–superconductor structure

Qing Lin He,[1*†] Lei Pan,[1†] Alexander L. Stern,[3] Edward C. Burks,[4] Xiaoyu Che,[1] Gen Yin,[1] Jing Wang,[5,6] Biao Lian,[6] Quan Zhou,[6] Eun Sang Choi,[7] Koichi Murata,[1] Xufeng Kou,[1,8*] Zhijie Chen,[4] Tianxiao Nie,[1] Qiming Shao,[1] Yabin Fan,[1] Shou-Cheng Zhang,[6*] Kai Liu,[4] Jing Xia,[3] Kang L. Wang[1,2*]

Majorana fermion is a hypothetical particle that is its own antiparticle. We report transport measurements that suggest the existence of one-dimensional chiral Majorana fermion modes in the hybrid system of a quantum anomalous Hall insulator thin film coupled with a superconductor. As the external magnetic field is swept, half-integer quantized conductance plateaus are observed at the locations of magnetization reversals, giving a distinct signature of the Majorana fermion modes. This transport signature is reproducible over many magnetic field sweeps and appears at different temperatures. This finding may open up an avenue to control Majorana fermions for implementing robust topological quantum computing.

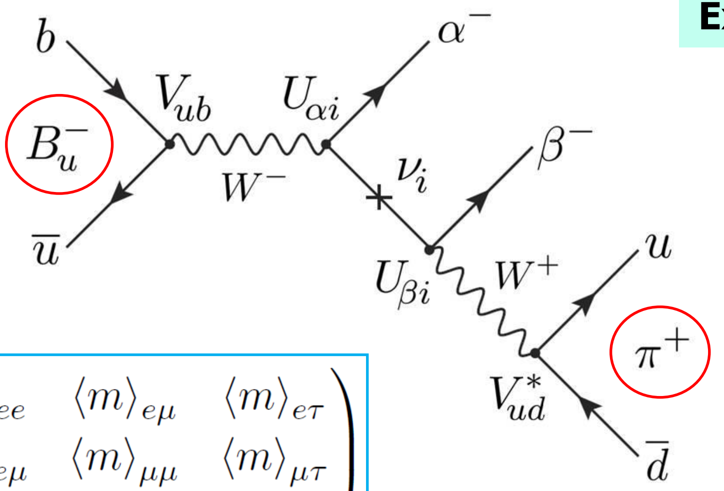
过去我们认为有粒子必有其反粒子，正如有天使必有魔鬼。但今天，我们找到了一个没有反粒子的粒子，一个只有天使，没有魔鬼的完美世界——张首晟

Editorial retraction 2022-11-18

Other LNV processes?

♦ **There are many LNV processes associated with heavy flavor decays but none of them observable?**

Example



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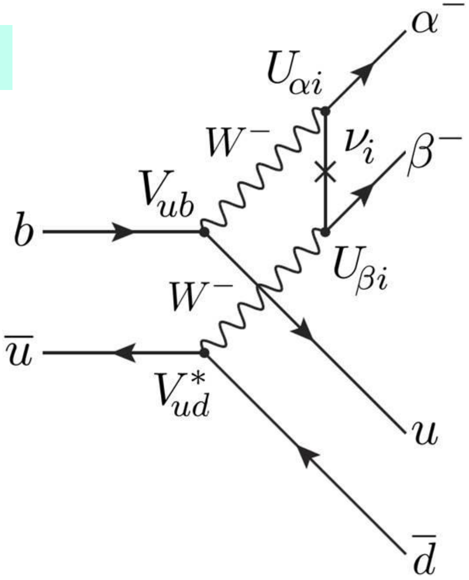


$$M_\nu = \begin{pmatrix} \langle m\rangle_{ee} & \langle m\rangle_{e\mu} & \langle m\rangle_{e\tau} \\ \langle m\rangle_{e\mu} & \langle m\rangle_{\mu\mu} & \langle m\rangle_{\mu\tau} \\ \langle m\rangle_{e\tau} & \langle m\rangle_{\mu\tau} & \langle m\rangle_{\tau\tau} \end{pmatrix}$$

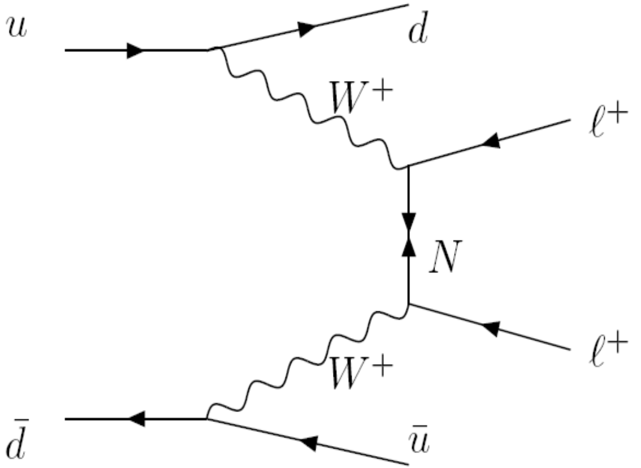
$$\Gamma(B_u^- \to \pi^+\alpha^-\beta^-) \propto \left|\langle m\rangle_{\alpha\beta}\right|^2 = \left|\sum_{i=1}^{3} (m_i U_{\alpha i} U_{\beta i})\right|^2$$

$$\mathcal{B}(B_u^- \to \pi^+ e^- e^-) < 2.3 \times 10^{-8} \ (\mathrm{CL} = 90\%)$$
$$\mathcal{B}(B_u^- \to \pi^+ e^- \mu^-) < 1.5 \times 10^{-7} \ (\mathrm{CL} = 90\%)$$
$$\mathcal{B}(B_u^- \to \pi^+ \mu^- \mu^-) < 4.0 \times 10^{-9} \ (\mathrm{CL} = 95\%)$$

♦ **Replacing the intermediate light Majorana neutrinos with the heavy ones, one can get new limits.**

Indirect way 5: collider signature?

♦ Heavy Majorana neutrinos are expected to *mediate* some LNV processes at a high energy and high luminosity collider, making it possible to search for possible signatures.

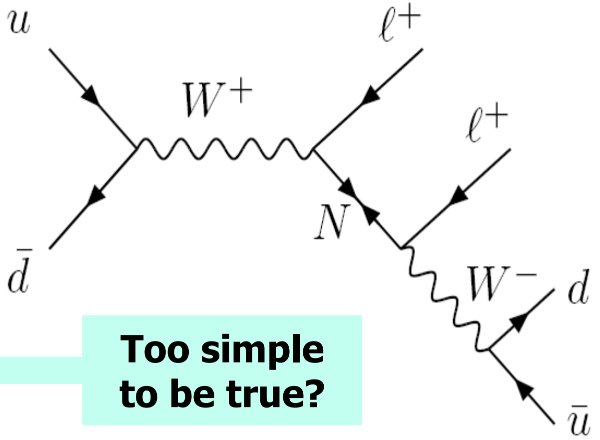


♦ An indirect collider signature at the Large Hadron Collider:

0ν2β-like: $pp \rightarrow W^{\pm}W^{\pm} \rightarrow \mu^{\pm}\mu^{\pm}jj$

$\Delta L = 2$ like-sign dilepton events

N-resonance: $pp \rightarrow W^{\pm} \rightarrow \mu^{\pm}N \rightarrow \mu^{\pm}\mu^{\pm}jj$



♦ Some experimental searches have been performed and all the results are negative. In particular, they have *little* to do with a real *seesaw* mechanism or with neutrino oscillations.

Too simple to be true?

Indirect way 6: leptogenesis

♦ **The flavor-dependent CP-violating asymmetries in the decays of three heavy Majorana neutrinos:**

$$\varepsilon_{j\alpha} \equiv \frac{\Gamma(N_j \to \ell_\alpha + H) - \Gamma(N_j \to \overline{\ell_\alpha} + \overline{H})}{\sum_\alpha \left[\Gamma(N_j \to \ell_\alpha + H) + \Gamma(N_j \to \overline{\ell_\alpha} + \overline{H})\right]}$$

M. Fukugita, T. Yanagida 1986
M. Luty 1992
L. Covi, E. Roulet, F. Vissani 1996

$$\simeq \frac{1}{8\pi\langle H\rangle^2 \sum_\beta |R_{\beta j}|^2} \sum_{k=4}^{6} \left\{ M_k^2 \, \mathrm{Im}\left[(R^*_{\alpha j} R_{\alpha k}) \sum_\beta \left[(R^*_{\beta j} R_{\beta k})\, \xi(x_{kj}) + (R_{\beta j} R^*_{\beta k})\, \zeta(x_{kj}) \right] \right] \right\}$$

♦ **Baryogenesis via leptogenesis in the early Universe:**

A net lepton number asymmetry

$$Y_{\rm L} \equiv \frac{n_{\rm L} - n_{\overline{\rm L}}}{s} = \frac{1}{g_*} \sum_{j,\alpha} \kappa_{j\alpha} \varepsilon_{j\alpha}$$

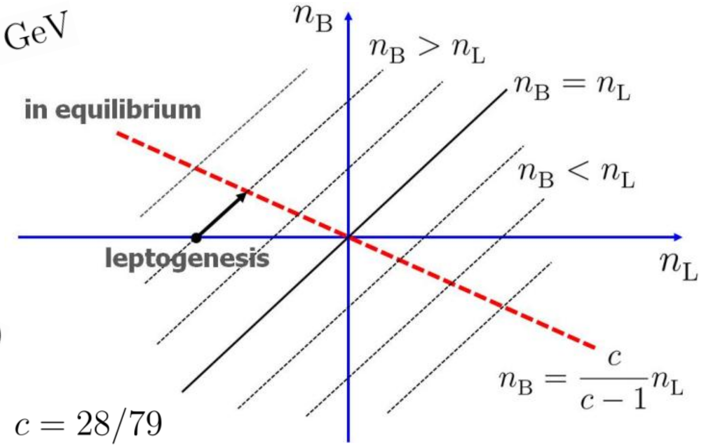
sphaleron-induced (B−L)-conserving process in thermal equilibrium

$$Y_{\rm B} \equiv \frac{n_{\rm B} - n_{\overline{\rm B}}}{s} = -cY_{\rm L}$$

A net baryon number asymmetry

$$10^2~\mathrm{GeV} \lesssim T \lesssim 10^{12}~\mathrm{GeV}$$

$$\Gamma(N_j \to \ell_\alpha + H) < \mathcal{H}(T = M_j)$$



It is analytically calculable!

♦ **CP-violating asymmetries of three heavy Majorana neutrino decays can all be expressed as linear combinations of the sines of the 6 original seesaw phase parameters:**

$$\varepsilon_{j\alpha} = \sum_{i=1}^{3} \left(C'_{\alpha i} \sin\alpha_i + C'_{\beta i} \sin\beta_i \right)$$

$$\varepsilon_{j} = \sum_{i=1}^{3} \left(C''_{\alpha i} \sin\alpha_i + C''_{\beta i} \sin\beta_i \right)$$

Flavor-independent CPV:

$$\varepsilon_j \equiv \varepsilon_{je} + \varepsilon_{j\mu} + \varepsilon_{j\tau}$$

$$\alpha_i \equiv \delta_{i4} - \delta_{i5} \, , \quad \beta_i \equiv \delta_{i5} - \delta_{i6}$$

♦ **The Jarlskog invariant of CP violation for neutrino oscillations can also be expressed as the linear combination of the sines of the 6 original seesaw phase parameters:**

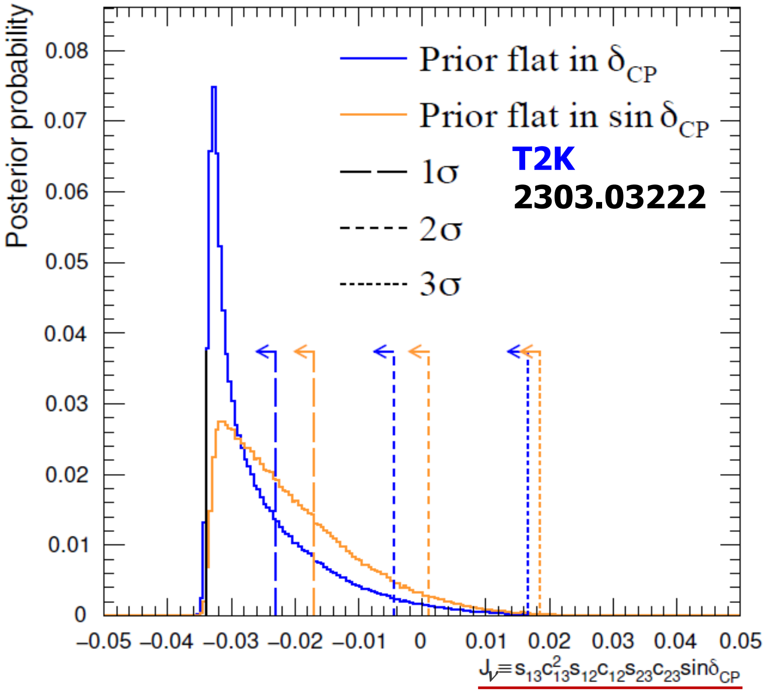
$$\mathcal{J}_\nu = \sum_{i=1}^{3} \left(C_{\alpha i} \sin\alpha_i + C_{\beta i} \sin\beta_i \right)$$

$$P(\nu_\mu \to \nu_e) = -4 \sum_{i<j} \left(\mathcal{R}_{ij} \sin^2 \frac{\Delta_{ji} L}{4E} \right) - 8 \mathcal{J}_\nu \prod_{i<j} \sin \frac{\Delta_{ji} L}{4E}$$

It is highly nontrivial to calculate the above coefficients in terms of the original seesaw parameters as first done in (ZZX, 2406.01142), but the analytical results are still too complicated.

Two kinds of CPV are correlated

♦ **3-flavor ν-oscillations are established and a 2σ hint for CPV is achieved.**



$$P(\nu_\mu \to \nu_e) = -4\sum_{i<j}\left(\mathcal{R}_{ij}\sin^2\frac{\Delta_{ji}L}{4E}\right) - 8\mathcal{J}_\nu \prod_{i<j}\sin\frac{\Delta_{ji}L}{4E}$$

direct connection

via

Seesaw plus leptogenesis

(ZZX, 2406.01142)

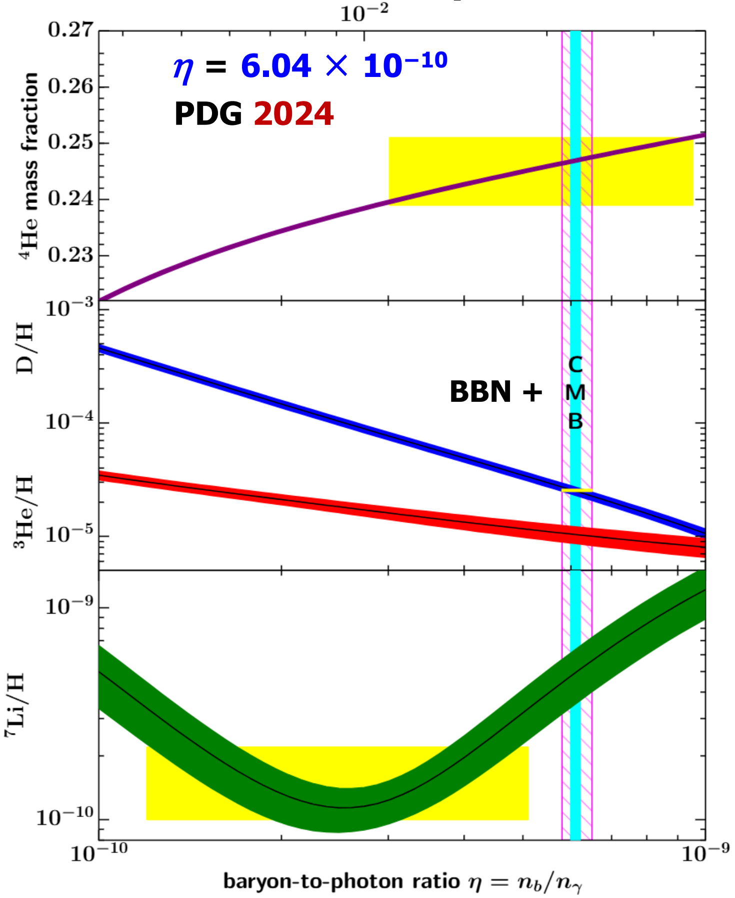
W. Buchmüller

M. Plümacher

NO direct link in general

hep-ph/9608308

♦ **Cosmic CPV is already established.**



Indirect way 7: neutrino oscillations

Nucl. Phys. B 1018 (2025) 117041

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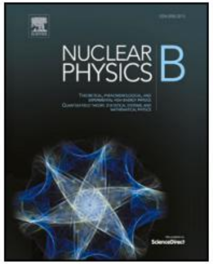


High Energy Physics – Phenomenology

Confronting the seesaw mechanism with neutrino oscillations: A general and explicit analytical bridge

Zhi-zhong Xing [a,c,d], Jing-yu Zhu [b,*]

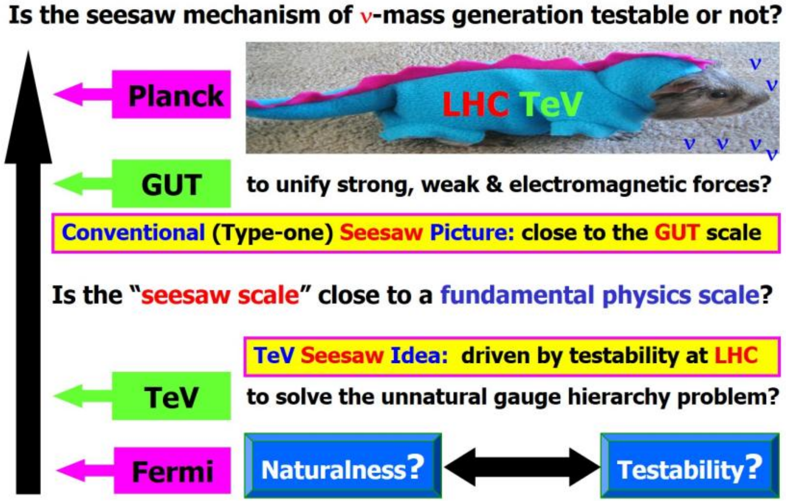
Current analytical results are still too lengthy to be useful. Some more effort is needed, to establish an easy connection between seesaw and data.

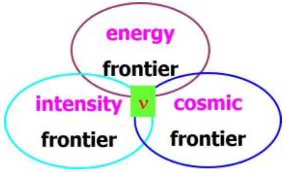
A B S T R A C T

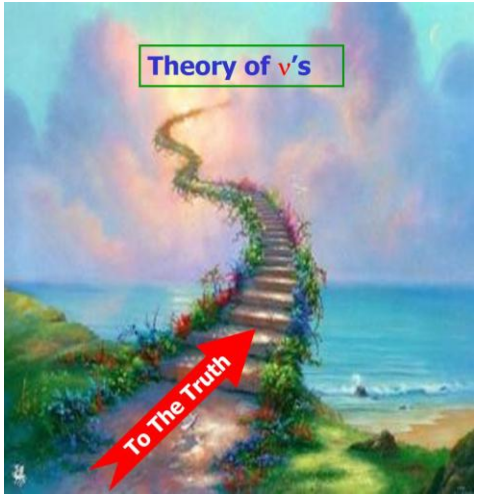
With the help of a full Euler-like block parametrization of the flavor structure for the canonical seesaw mechanism, we present the *first* general and explicit analytical calculations of the two neutrino mass-squared differences, three flavor mixing angles and the effective Dirac CP-violating phase responsible for the primary behaviors of neutrino oscillations. Such model-independent results will pave the way for testing the seesaw mechanism at low energies.

A brief summary

♦ **The question that I asked in my plenary talk given at "ICHEP 2008" in Philadelphia remains open?**







Today's opinion

- ♦ **Direct discovery of heavy Majorana DoFs: very challenging**
- ♦ **Constraints from cLFV processes: highly desired**
- ♦ **Constraints from LNV processes: highly wanted**
- ♦ **Constraints from neutrino oscillations: a big deal**
- ♦ **Constraints from other approaches: encouraging**

OUTLINE

- Why neutrinos and why massless?
- Why Majorana and why not Dirac?
- Possible ways to test the seesaw?
- There is a flavor symmetry behind
- Inconclusive remarks

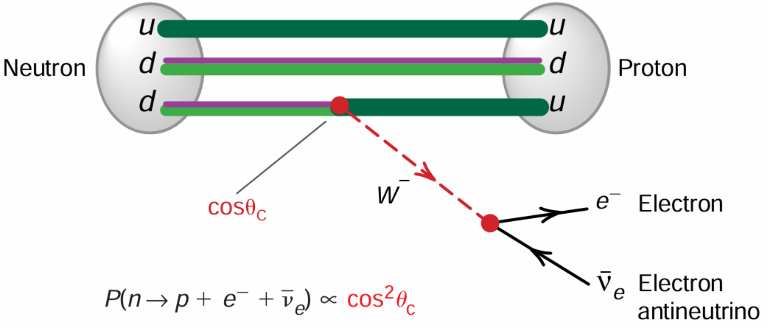
The seed of flavor mixing

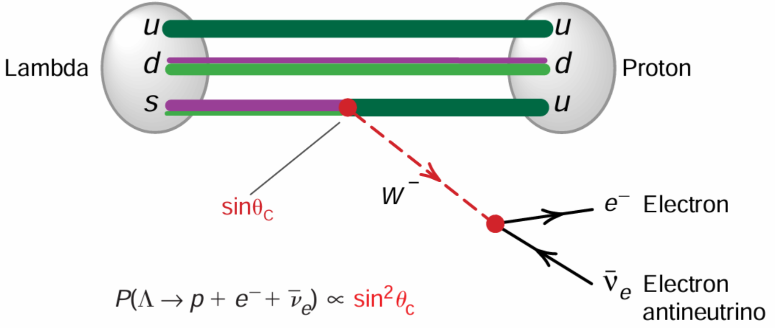
M. Gell-Mann, M. Levy: the axial current in beta decay (Nuovo Cim. 16, 705, 1960):

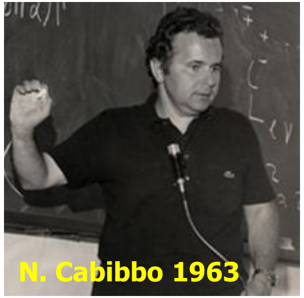
(*) *Note added in proof.* – Should this discrepancy be real, it would probably indicate a total or partial failure of the conserved vector current idea. It might also mean, however, that the current is conserved but with $G/G_\mu < 1$. Such a situation is consistent with universality if we consider the vector current for $\Delta S = 0$ and $\Delta S = 1$ together to be something like:

$$GV_\alpha + GV_\alpha^{(\Delta s=1)} = G_\mu \bar{p}\gamma_\alpha(n + \varepsilon\Lambda)(1+\varepsilon^2)^{-\frac{1}{2}} + \dots,$$

and likewise for the axial vector current. If $(1+\varepsilon^2)^{-\frac{1}{2}} = 0.97$, then $\varepsilon^2 = .06$, which is of the right order of magnitude for explaining the low rate of β decay of the Λ particle. There is, of course, a renormalization factor for that decay, so we cannot be sure







The only weak phase in the SM

In QFTs: • non-observable phase → a possible symmetry; • observable phase → symmetry breaking.

In 1973, M. Kobayashi and T. Maskawa proposed a mechanism of CP violation in the SM.

Progress of Theoretical Physics, Vol. 49, No. 2, February 1973

CP-Violation in the Renormalizable Theory of Weak Interaction

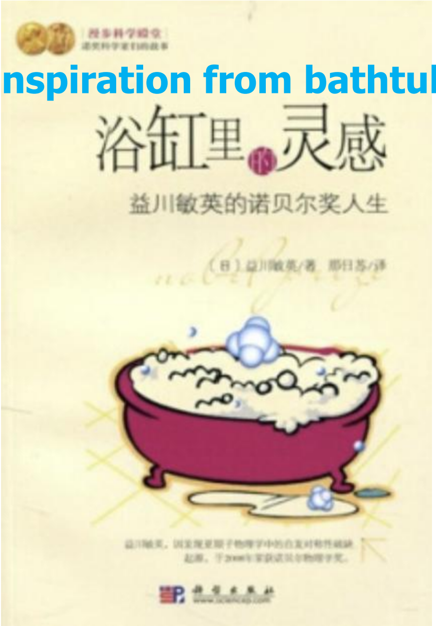
Makoto KOBAYASHI and Toshihide MASKAWA

Department of Physics, Kyoto University, Kyoto

(Received September 1, 1972)

In a framework of the renormalizable theory of weak interaction, problems of *CP*-violation are studied. It is concluded that no realistic models of *CP*-violation exist in the quartet scheme without introducing any other new fields. Some possible models of *CP*-violation are also discussed.

Inspiration from bathtub



It is the nontrivial KM phase that determines all the phenomena of CP violation in the quark sector.

This is the only phase parameter in particle physics that has so far been observed. More to be seen?

Why flavor mixing + CP violation?

★ **Reason 1: the fermion fields interact, simultaneously but in different ways, with the Higgs and gauge fields.**

★ **Reason 2: the fermions have three different families.**

} **the Kobayashi-Maskawa mechanism**

★ **In the flavor basis, quark masses, flavor mixing and CP violation originate from the complex and non-diagonal Yukawa interactions.**

$$C_q \equiv \mathrm{i}\left[M_{\mathrm{u}}M_{\mathrm{u}}^{\dagger}, M_{\mathrm{d}}M_{\mathrm{d}}^{\dagger}\right] \longrightarrow \det C_q = -2\mathcal{J}_q\left(m_u^2 - m_c^2\right)\left(m_c^2 - m_t^2\right)\left(m_t^2 - m_u^2\right)\left(m_d^2 - m_s^2\right)\left(m_s^2 - m_b^2\right)\left(m_b^2 - m_d^2\right)$$

Jarlskog invariant: $\mathcal{J}_q = \frac{1}{8}\sin 2\vartheta_{12}\sin 2\vartheta_{13}\sin 2\vartheta_{23}\cos\vartheta_{13}\sin\delta_q$

Similar to QM?

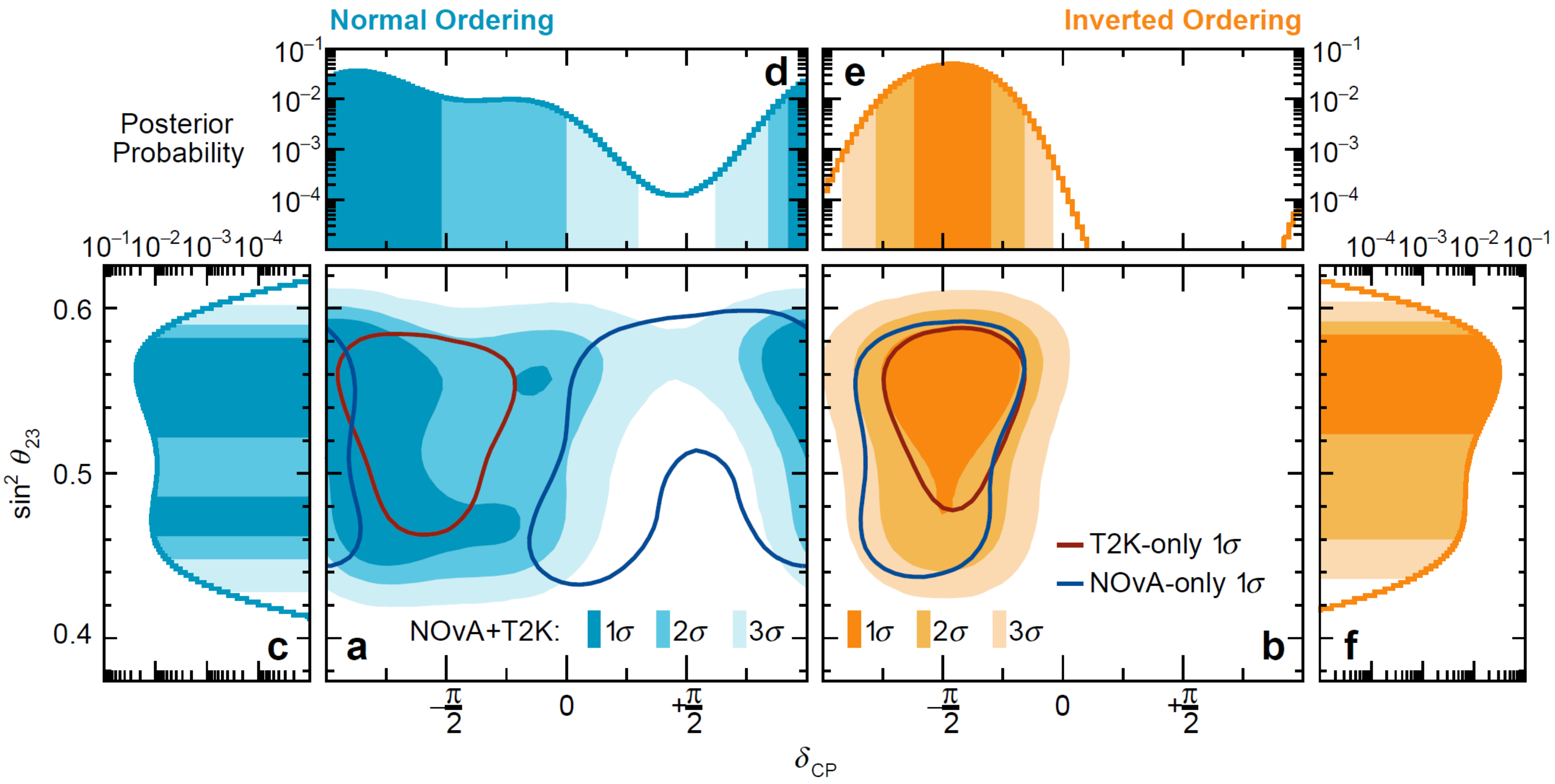
- **W. Heisenberg, ZPC 33 (1925) 879**
- **M. Born, P. Jordan, ZPC 34 (1925) 858**
- **M. Born, W. Heisenberg, P. Jordan, ZPC 35 (1926) 557**

$$\left[\hat{x}(t), \hat{p}(t)\right] = \mathrm{i}\hbar$$

★ **In the mass basis, flavor mixing and CP violation are described by a 3 × 3 unitary matrix with an irremovable KM phase in weak charged-current interactions.** **Non-unitarity in the seesaw case!**

Hint from T2K + NOνA

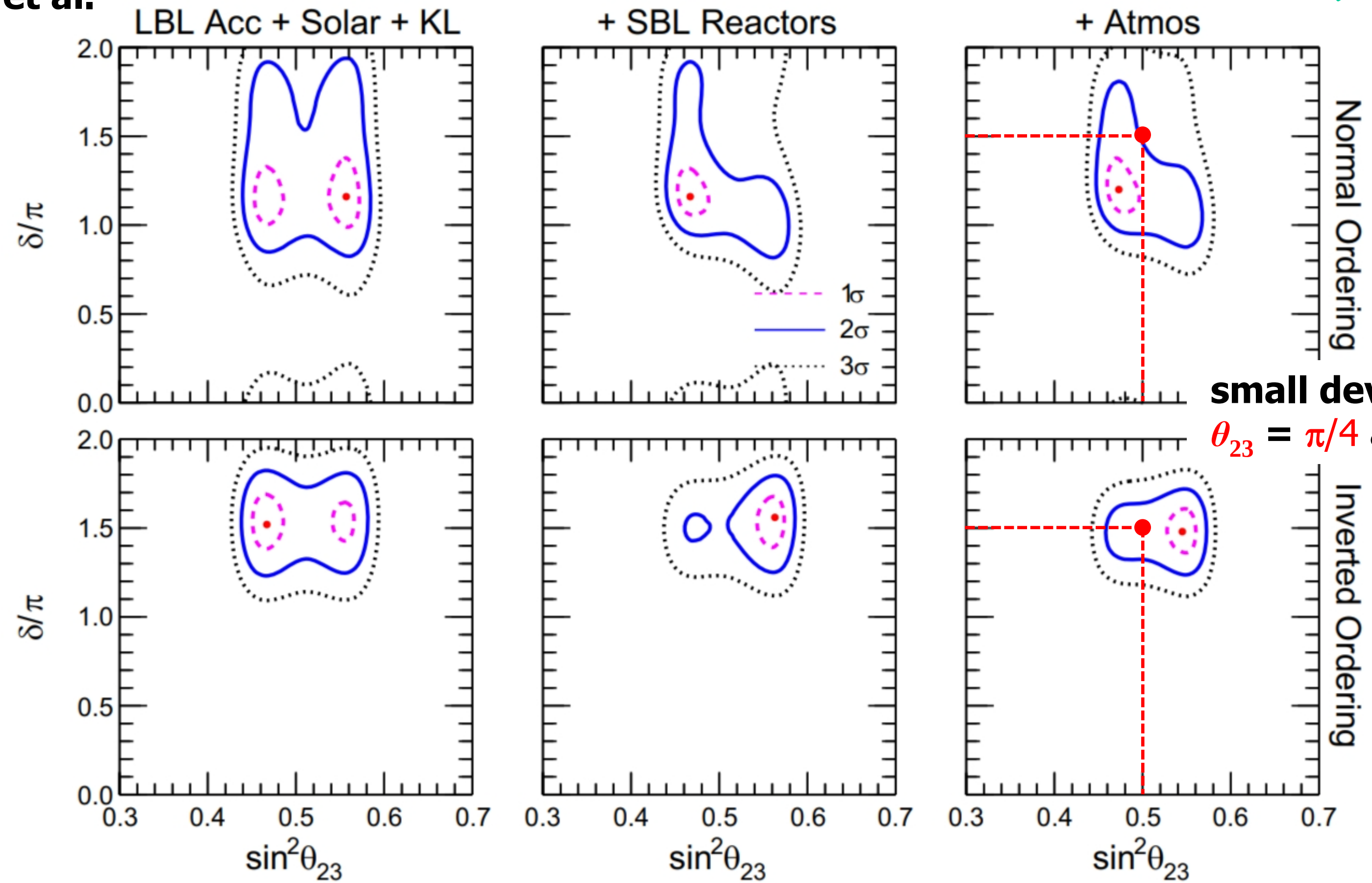
- The latest joint T2K and NOνA analysis (2510.19888, appearing on 2025/10/24) gives the result:



- small deviations from $\theta_{23} = \pi/4$ and $\delta = -\pi/2$?

Hint from a global fit

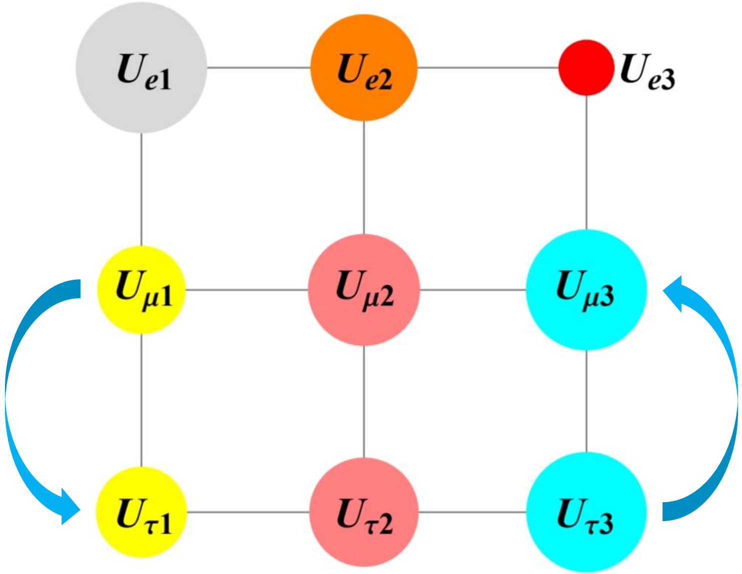
♦ F. Capozzi et al. 2503.07752



small deviations from $\theta_{23} = \pi/4$ and $\delta = -\pi/2$?

A flavor symmetry behind?

♦ **9 moduli of the PMNS matrix elements constrained from data at the 3σ level:**



the area of each circle = an element’s modulus

♦ **The standard parametrization of the PMNS matrix with 3 Euler-like mixing angles and 3 CPV phases:**

PMNS =  

$$U_0 = \begin{pmatrix} c_{12}c_{13} & \hat{s}^*_{12}c_{13} & \hat{s}^*_{13} \\ -\hat{s}_{12}c_{23} - c_{12}\hat{s}_{13}\hat{s}^*_{23} & c_{12}c_{23} - \hat{s}^*_{12}\hat{s}_{13}\hat{s}^*_{23} & c_{13}\hat{s}^*_{23} \\ \hat{s}_{12}\hat{s}_{23} - c_{12}\hat{s}_{13}c_{23} & -c_{12}\hat{s}_{23} - \hat{s}^*_{12}\hat{s}_{13}c_{23} & c_{13}c_{23} \end{pmatrix}$$

$$\theta_{23} \sim \pi/4 \oplus \begin{cases} \theta_{13} \ll 1 \\ \text{or} \\ \delta \equiv \delta_{13} - \delta_{12} - \delta_{23} \sim \pm\pi/2 \end{cases}$$

$$|U_{\mu i}| \simeq |U_{\tau i}| \quad (i = 1, 2, 3)$$

P. Harrison, W. Scott (2002):

mu-tau reflection symmetry

with both $\theta_{23} = \pi/4$ & $\delta = -\pi/2$

We are on the right track

What's mu-tau reflection?

It is a **working flavor symmetry** requiring the effective **Majorana** neutrino mass term to be invariant under the transformations of left-handed neutrino fields [ZZX, Z.H. Zhao, 1512.04207]:

$$-\mathcal{L}_{\rm mass} = \frac{1}{2}\overline{\nu_{\rm L}}\, M_\nu \,(\nu_{\rm L})^c + {\rm h.c.} \quad \longleftarrow \quad \nu_{e{\rm L}} \to (\nu_{e{\rm L}})^c\,, \quad \nu_{\mu{\rm L}} \to (\nu_{\tau{\rm L}})^c\,, \quad \nu_{\tau{\rm L}} \to (\nu_{\mu{\rm L}})^c$$

◆ **traditional CP transformation**

$$(t, \mathbf{x}) \longrightarrow (t, -\mathbf{x})$$

$$\begin{cases} \nu_{e{\rm L}} \longrightarrow (\nu_{e{\rm L}})^c \\ \nu_{\mu{\rm L}} \longrightarrow (\nu_{\mu{\rm L}})^c \\ \nu_{\tau{\rm L}} \longrightarrow (\nu_{\tau{\rm L}})^c \end{cases}$$

◆ **mu-tau-interchanging CP transformation**

$$(t, \mathbf{x}) \longrightarrow (t, -\mathbf{x})$$

$$\begin{cases} \nu_{e{\rm L}} \longrightarrow (\nu_{e{\rm L}})^c \\ \nu_{\mu{\rm L}} \longrightarrow (\nu_{\tau{\rm L}})^c \\ \nu_{\tau{\rm L}} \longrightarrow (\nu_{\mu{\rm L}})^c \end{cases}$$

Invariance: $M_\nu = M_\nu^*$ **CP conserving** $\qquad M_\nu = \mathcal{P} M_\nu^* \mathcal{P}$ **CP violating**

Constraints on the flavor structure of three Majorana neutrinos:

$$M_\nu = \begin{pmatrix} C & D & D^* \\ D & A & B \\ D^* & B & A^* \end{pmatrix}$$

$\nu_e \quad \nu_\mu \leftrightarrow \nu_\tau^c$

$$\mathcal{P} = \mathcal{P}^T = \mathcal{P}^\dagger = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

mu-tau permutation

$$\theta_{23} = \pi/4\,, \ \delta = \pm\,\pi/2$$

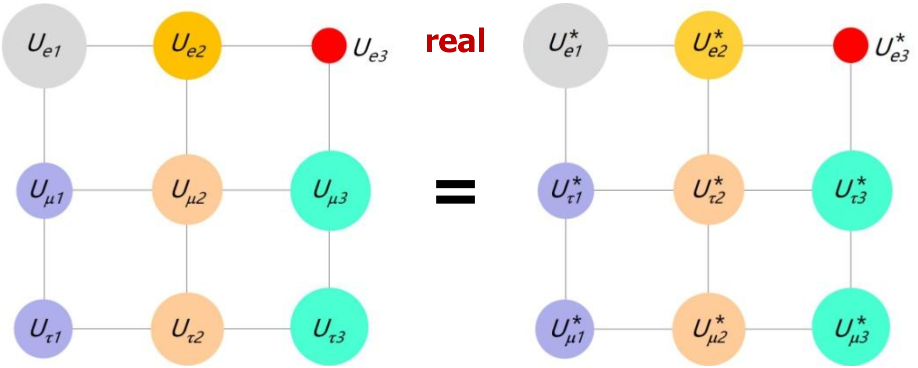
A reverse approach

- **Different from previous works, here let us start purely from the PMNS matrix constrained by data:**

A data-driven conjecture: $U = \mathcal{P} U^* \zeta$

$$\mathcal{P} = \mathcal{P}^T = \mathcal{P}^\dagger = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\zeta = \mathrm{Diag}\{\eta_1, \eta_2, \eta_3\} \text{ with } \eta_i = \pm 1$$



- **In the basis where flavor states of charged leptons are identified with their mass states, we have the effective Majorana neutrino mass matrix**

$$M_\nu = U D_\nu U^T = \mathcal{P} U^* \zeta D_\nu \zeta U^\dagger \mathcal{P} = \mathcal{P} \left(U D_\nu U^T \right)^* \mathcal{P} = \mathcal{P} M_\nu^* \mathcal{P}$$

$$D_\nu \equiv \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

Substitute this into the mass term:

$$-\mathcal{L}_{\rm mass} = \frac{1}{2} \overline{\nu_{\rm L}}\, M_\nu \,(\nu_{\rm L})^c + {\rm h.c.}$$

$$-\mathcal{L}'_{\rm mass} = \frac{1}{2} \overline{\nu_{\rm L}} \,(\mathcal{P} M_\nu^* \mathcal{P})\, (\nu_{\rm L})^c + {\rm h.c.} = \frac{1}{2} \overline{[\mathcal{P}(\nu_{\rm L})^c]}\, M_\nu \,[\mathcal{P}\nu_{\rm L}] + {\rm h.c.}$$

Then the invariance $\mathcal{L}'_{\rm mass} = \mathcal{L}_{\rm mass}$ leads us to the μ-τ reflection transformation $\nu_{\rm L} \to \mathcal{P}(\nu_{\rm L})^c$. QED

Go across the seesaw bridge

♦ **Non-unitarity** of the **PMNS** matrix has been constrained to be **≤ 0.1 %**. So even in the presence of tiny unitarity violation, one may still make the conjecture:

$$|U_{\mu i}| = |U_{\tau i}| \quad i = 1, 2, 3 \longrightarrow U = \mathcal{P} U^* \zeta$$

$$D_N \equiv \mathrm{Diag}\{M_4, M_5, M_6\}$$

seesaw duality symmetry:

$$U D_\nu U^T + R D_N R^T = \mathbf{0}$$

symmetry dictates texture:

$$D_\nu \equiv \mathrm{Diag}\{m_1, m_2, m_3\}$$

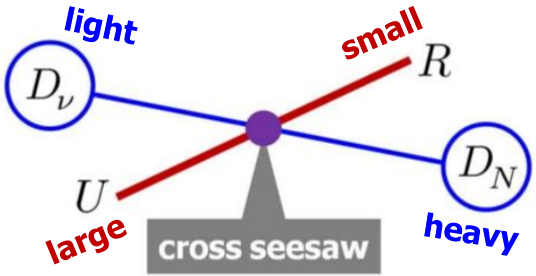
novel prediction

$$R = \mathcal{P} R^* \zeta' \longrightarrow |R_{\mu j}| = |R_{\tau j}| \quad j = 4, 5, 6$$

♦ **The top-down approach works in the same way —— the seesaw bridge helps *transmit* a potential μ-τ reflection symmetry of *R* to the active neutrino sector, leading to a μ-τ symmetry of *U*:**

The active (light) sector:

- **Naturally small neutrino masses**
- **Emergently large flavor mixing**



The sterile (heavy) sector:

- **Extremely tiny Yukawa couplings**
- **Possible μ-τ reflection symmetry**

How right-handed fields transform

♦ Let us consider the neutrino mass term in the **seesaw** mechanism: $M_{\rm D} \equiv Y_\nu \langle H \rangle$

$$-\mathcal{L}_\nu = \overline{\ell_{\rm L}} Y_\nu \widetilde{H} N_{\rm R} + \frac{1}{2} \overline{(N_{\rm R})^c} M_{\rm R} N_{\rm R} + {\rm h.c.}$$

SSB →

$$-\mathcal{L}'_\nu = \frac{1}{2} \overline{[\nu_{\rm L} \ \ (N_{\rm R})^c]} \begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R} \end{pmatrix} \begin{bmatrix} (\nu_{\rm L})^c \\ N_{\rm R} \end{bmatrix} + {\rm h.c.}$$

Diagonalizing the 6 × 6 neutrino mass matrix:

$$\begin{pmatrix} U & R \\ S & Q \end{pmatrix}^\dagger \begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R} \end{pmatrix} \begin{pmatrix} U & R \\ S & Q \end{pmatrix}^* = \begin{pmatrix} D_\nu & \mathbf{0} \\ \mathbf{0} & D_N \end{pmatrix}$$

exact seesaw →

$$U D_\nu U^T + R D_N R^T = \mathbf{0}$$

Unitarity:

$$\begin{cases} UU^\dagger + RR^\dagger = SS^\dagger + QQ^\dagger = I \\ U^\dagger U + S^\dagger S = R^\dagger R + Q^\dagger Q = I \\ US^\dagger + RQ^\dagger = U^\dagger R + S^\dagger Q = \mathbf{0} \end{cases}$$

$$U = \mathcal{P} U^* \zeta \longrightarrow R = \mathcal{P} R^* \zeta'$$

$$S = \mathcal{T} S^* \zeta \qquad Q = \mathcal{T} Q^* \zeta'$$

$\mathcal{T}$ = arbitrary unitary transformation

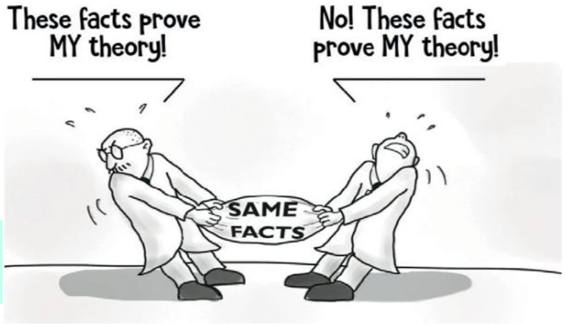
$$M_{\rm D} = \mathcal{P} M_{\rm D}^* \mathcal{T} \ , \quad M_{\rm R} = \mathcal{T}^T M_{\rm R}^* \mathcal{T}$$

♦ Substitute these into the above neutrino mass term and require it to be invariant, we get transformations:

$$\nu_{\rm L} \to \mathcal{P} (\nu_{\rm L})^c \ , \quad N_{\rm R} \to \mathcal{T}^* (N_{\rm R})^c$$

Comments on model building

♦ **Perhaps 1000 models based on *flavor symmetries* have been built in the past three decades, to understand why lepton flavor mixing is as observed. Seesaws are needed in most cases.**



S_3, S_4, A_4, A_5, D_4, D_7, T_7, T′, Δ(27), Δ(48), …

$U(1)_F$, $SU(2)_F$, … modular, …

Big model?

Small model?

♦ **In this way one often proceeds with**

- **a guiding principle (TH) or experimental hints (PH)**
- **a toolbox to make the model give something fine**
- **a dustbin to collect and hide some ugly things**

♦ **A symmetry implies that *behind it* there is something *unobservable*, but a flavor symmetry must be broken to makes something observable. Symmetry breaking is highly nontrivial.**

The bottom line is to fit data —— a clear physical picture and not many free parameters?

The review papers since 2000: ZZX, 1909.09610 (PR 2020); F. Feruglio, A. Romanino, 1912.06028 (RMP 2021); ZZX, 2210.11922 (RPP 2023); G.J. Ding, S.F. King, 2311.09282 (RPP 2024); G.J. Ding, J.W.F. Valle, 2402.16963 (PR 2025)

Today's best seller: modular symmetry

- Many modular invariant model building exercises (G. Altarelli, F. Feruglio 2006; F. Feruglio 2017).

- Orbifold compactification: 10D string theory → 4D SM + 3 copies of 2D torus.

- A complex modulus τ is enough for describing the shape of torus. A modular invariant super-potential gives rise to the modular forms of Yukawa coupling matrices which depend on τ.

- The seesaw mechanism is almost always invoked.

Comment A: physical meaning of the complex modular parameter τ is unclear?

Comment B: flavor textures are not transparent due to a *nonlinear* realization of modular symmetry, and hence a careful numerical fitting has to be done?

Comment C: no good reason for a strong mass hierarchy of *charged fermions*?

- In contrast, the conventional (*discrete*) flavor symmetries can linearly predict flavor mixing with CG coefficients, and thus more transparent in physics. *None is simple!*

A favorite flavor mixing pattern?

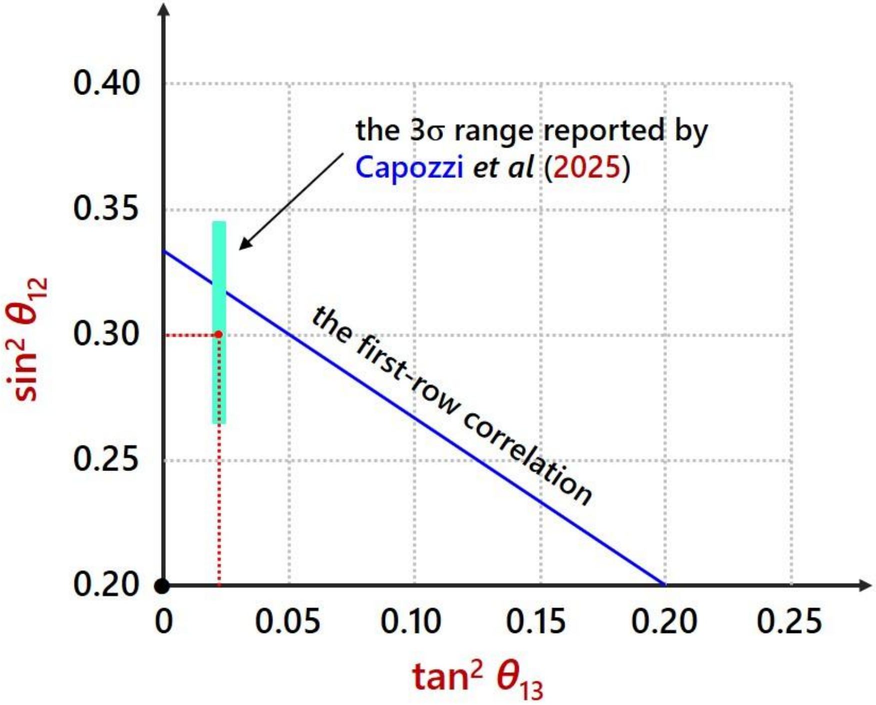
♦ **To get a balance between model building (easy) and data fitting (good), I bet on the TM1 pattern of lepton flavor mixing (ZZX, S. Zhou, hep-ph/0607302; C.S. Lam, hep-ph/0611017; many models):**

$$U'_* = \begin{pmatrix} \dfrac{2}{\sqrt{6}} & \dfrac{c_*}{\sqrt{3}} & \dfrac{s_*}{\sqrt{3}} e^{-i\phi} \\ -\dfrac{1}{\sqrt{6}} & \dfrac{c_*}{\sqrt{3}} - \dfrac{s_*}{\sqrt{2}} e^{i\phi} & \dfrac{c_*}{\sqrt{2}} + \dfrac{s_*}{\sqrt{3}} e^{-i\phi} \\ \dfrac{1}{\sqrt{6}} & -\dfrac{c_*}{\sqrt{3}} - \dfrac{s_*}{\sqrt{2}} e^{i\phi} & \dfrac{c_*}{\sqrt{2}} - \dfrac{s_*}{\sqrt{3}} e^{-i\phi} \end{pmatrix} P_\nu$$

$$P_\nu = \mathrm{Diag}\left\{e^{i\rho}, e^{i\sigma}, 1\right\}$$

which possesses a partial mu-tau symmetry but predicts a striking first-row correlation:

$$\sin^2\theta_{12} = \frac{1}{3}\left(1 - 2\tan^2\theta_{13}\right)$$



♦ **The forthcoming JUNO precision measurements, combined with the Daya Bay precision data, will directly test this first-row correlation (ZZX, 2510.17583).**

How about an inverse seesaw?

♦ The inverse seesaw framework (D. Wyler, L. Wolfenstein 1983; R. Mohapatra, J.W.F. Valle, 1986):

$$-\mathcal{L}'_{\rm ss} = \overline{\ell_{\rm L}}\,\widehat{Y}_l H l_{\rm R} + \overline{\ell_{\rm L}}\,Y_\nu \widetilde{H} N_{\rm R} + \overline{(N_{\rm R})^c}\,Y_S \Phi S_{\rm R} + \frac{1}{2}\overline{(S_{\rm R})^c}\,\hat{\mu} S_{\rm R} + {\rm h.c.}$$

$$= \overline{l_{\rm L}}\,\widehat{Y}_l l_{\rm R}\phi^0 + \frac{1}{2}\overline{\left[\nu_{\rm L} \quad (N_{\rm R})^c \quad (S_{\rm R})^c\right]}\begin{pmatrix} 0 & Y_\nu \phi^{0*} & 0 \\ Y_\nu^T \phi^{0*} & 0 & Y_S \Phi \\ 0 & Y_S^T \Phi & \hat{\mu} \end{pmatrix}\begin{bmatrix} (\nu_{\rm L})^c \\ N_{\rm R} \\ S_{\rm R} \end{bmatrix}$$

$$+ \overline{\nu_{\rm L}}\,\widehat{Y}_l l_{\rm R}\phi^+ - \overline{l_{\rm L}}\,Y_\nu N_{\rm R}\phi^- + {\rm h.c.}\,,$$

- To lower the seesaw scale.
- Cost: many parameters.
- Gain: many papers?

♦ Diagonalization:

$$\mathbb{U}'^\dagger \begin{pmatrix} 0 & Y_\nu \phi^{0*} & 0 \\ Y_\nu^T \phi^{0*} & 0 & Y_S \Phi \\ 0 & Y_S^T \Phi & \hat{\mu} \end{pmatrix} \mathbb{U}'^* = \begin{pmatrix} D_\nu & 0 & 0 \\ 0 & D_N & 0 \\ 0 & 0 & D_S \end{pmatrix}$$

$$D_\nu = \{m_1, m_2, m_3\}$$
$$D_N = \{M_4, M_5, M_6\}$$
$$D_S = \{M'_7, M'_8, M'_9\}$$

Weak CC interactions:

$$-\mathcal{L}'_{\rm cc} = \frac{g}{\sqrt{2}}\,\overline{(e \quad \mu \quad \tau)_{\rm L}}\,\gamma^\mu \left[U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} + R \begin{pmatrix} N_4 \\ N_5 \\ N_6 \end{pmatrix}_{\rm L} + R' \begin{pmatrix} N'_7 \\ N'_8 \\ N'_9 \end{pmatrix}_{\rm L} \right] W^-_\mu + {\rm h.c.}$$

$$U \equiv A_2 A_1 U_0$$
$$R \equiv A_2 R_1$$
$$R' \equiv R_2$$

fine cancellation

♦ The exact inverse seesaw relation:

$$U D_\nu U^T = ({\rm i}R)\,D_N\,({\rm i}R)^T + ({\rm i}R')\,D_S\,({\rm i}R')^T$$

H.C. Han, ZZX, 2110.12705

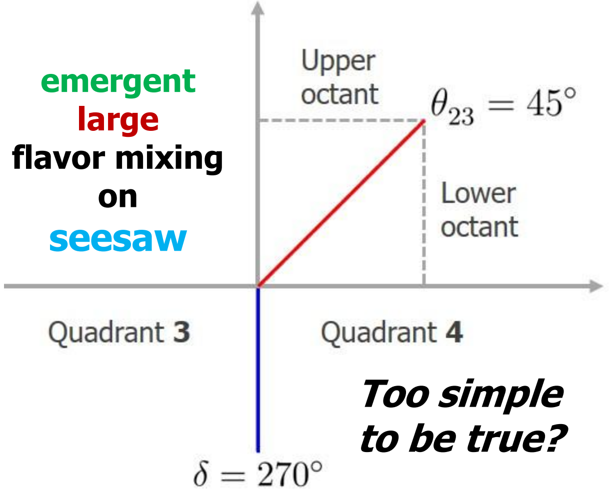
A brief summary

- 30 years ago, **H. Fritzsch** and **I** proposed an **S(3)-symmetry**-driven lepton mass **ansatz**, predicting the **1st** (**2 large** + **1 small**)-angle flavor mixing pattern (hep-ph/9509389, published in **April 1996**):

$$U = \begin{pmatrix} \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} & 0 \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{6}} & -\sqrt{\frac{2}{3}} \\ \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} \end{pmatrix} + {\rm i}\sqrt{\frac{m_e}{m_\mu}} \begin{pmatrix} \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{6}} & -\sqrt{\frac{2}{3}} \\ \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} & 0 \\ 0 & 0 & 0 \end{pmatrix} + \frac{m_\mu}{m_\tau} \begin{pmatrix} 0 & 0 & 0 \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{6}} \\ -\sqrt{\frac{1}{12}} & -\sqrt{\frac{1}{12}} & \sqrt{\frac{1}{3}} \end{pmatrix} \rightarrow \begin{array}{l} \theta_{12} \simeq 42^\circ \\ \theta_{13} \simeq 4^\circ \\ \theta_{23} \simeq 52^\circ \\ \delta \simeq \pm 90^\circ \end{array}$$

In **June 1998**, the **Super-K** data on solar + atmospheric neutrinos hinted at $\theta_{12} \simeq \theta_{23} \simeq 45^\circ$. **New Era**!

- Today we bet on a **data-driven** μ-τ **reflection symmetry**, and have tried many simple or complicated flavor groups for model building. Are some theorists' tastes exotic?



- Though it is always fine to follow a **bottom-up** approach towards understanding the *flavor structures* of **Majorana** and **Dirac** fermions, I believe that a true solution to flavor issues must be **top-down**.

  ***Theory** is **King** in this regard, not **data**.*

OUTLINE

- Why neutrinos and why massless?
- Why Majorana and why not Dirac?
- Possible ways to test the seesaw?
- There is a flavor symmetry behind
- Inconclusive remarks

No new physics at TeV?

♦ **Many theorists like S. Dimopoulos conjectured an emergence of new physics at TeV 35 years ago.**

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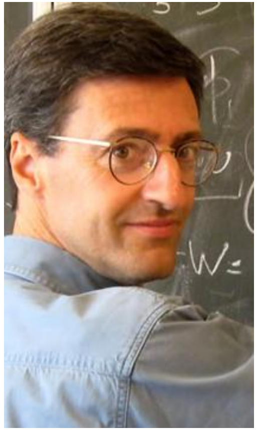
LHC, SSC and the universe

Savas Dimopoulos [1,2]

CERN, CH-1211 Geneva 23, Switzerland
and Boston University, Boston, MA 02215, USA

$$\mathrm{TeV} \simeq \sqrt{M_{\mathrm{Pl}} \times 2.7\ \mathrm{K}}$$



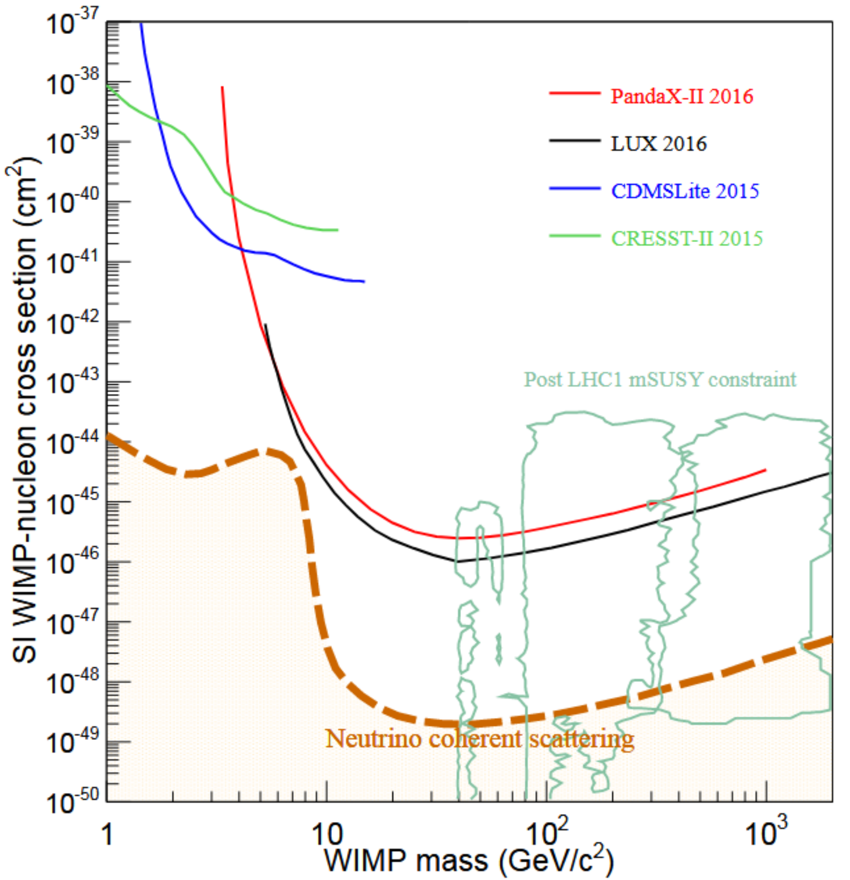
Received 12 June 1990

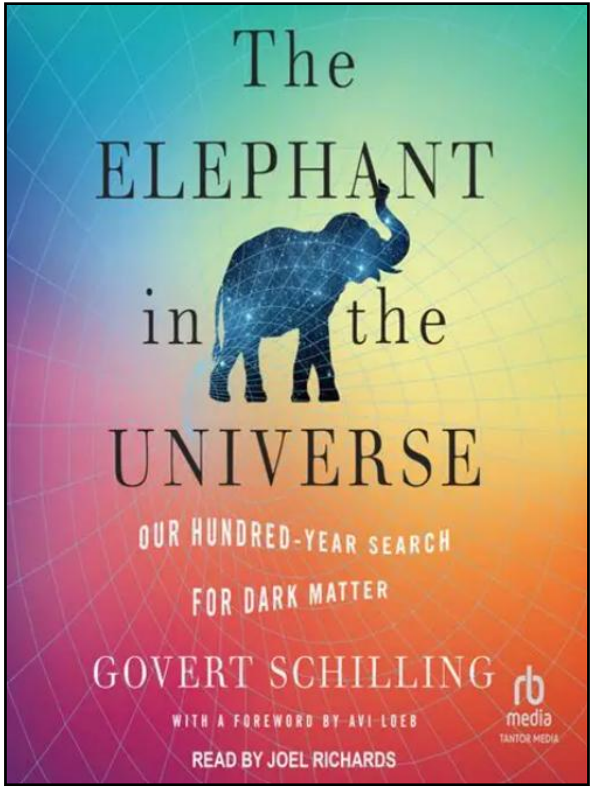
The geometric mean of the Planck mass and the 2.7 K background temperature – numerically equal to about a TeV – is the *maximum* mass that any cosmologically stable perturbatively coupled elementary particle can have or else the density of the universe exceeds its critical value. Thus, the TeV scale is cosmologically significant for reasons unrelated to the scale of electroweak symmetry breaking; it would persist even if the masses of the W and Z vanished. This implies that the TeV scale emerges cosmologically in many extensions of the standard model involving new particles and forces. We derive, for example, *upper* limits

♦ **Naturalness of the SM was also expected to point to TeV-scale new physics (G. Giudice: *Naturally Speaking: The Naturalness Criterion and Physics at the LHC*, 0801.2562), but nothing was seen.**

No success in dark matter search

♦ **Cold dark matter is highly expected to exist, but none of the *direct* searches has been successful.**



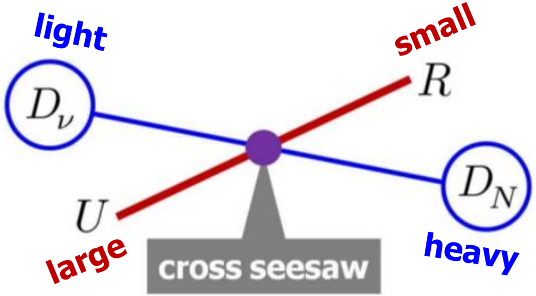


♦ **Does the two-parameter analysis really make sense?**

Neutrinos: a big hope ahead

♦ Only neutrino oscillations (and gravitational waves) have successfully been established thanks to quantum interference. We neutrino physicists are lucky in this regard.

♦ But we still have a long way to go, especially on the TH side. In this talk, I have tried to *convince* you and myself that the Minkowski mechanism of neutrino mass generation is most likely, and that there should be a simple flavor symmetry behind the observed pattern of lepton flavor mixing.



♦ I believe that you are not fully convinced, nor myself. How could we do better in the near future?

Many Thanks for Your Comments and Criticisms