Flavored QCD axion and Modular invariance

Collaboration with Prof. Eung Jin Chun
appear soon in arXiv 2511.xxxxx

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Standard Model as a low energy effective theory

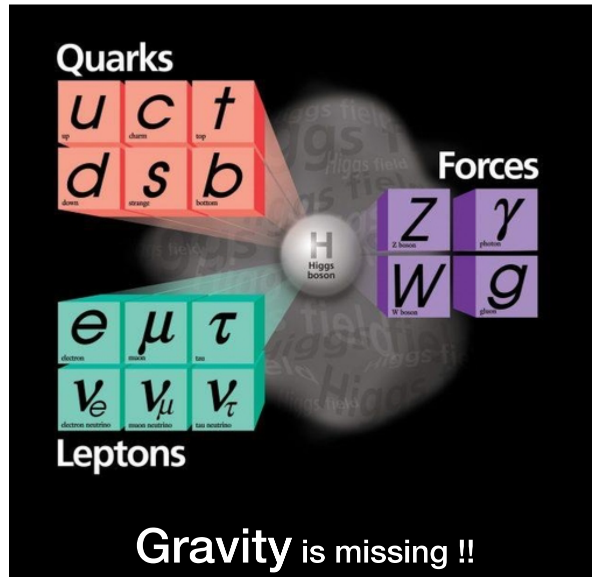
Symmetries -

Lorentz invariance

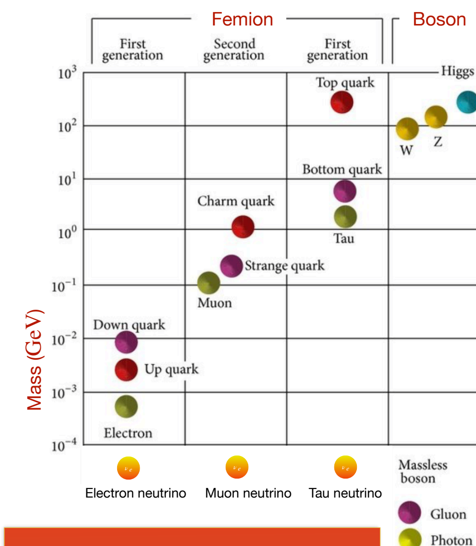
$SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry

Space-time symmetry (P & CP)

Absence of $SU(N)$ & $U(1)$ anomalies



SM Particle Mass



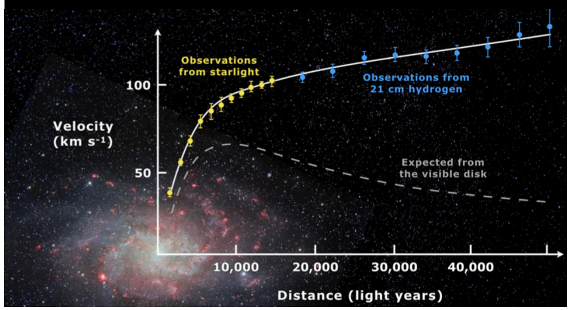
Neutrino mass = 0

Experimentally, excluding the Higgs mechanism, completed in the 70s and 80s

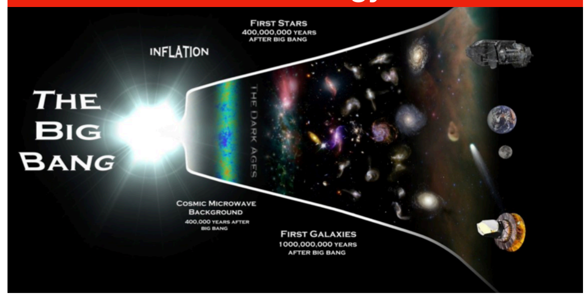
The Higgs mechanism Granting mass
to other fundamental particles
The discovery of the Higgs boson in 2012

Things that can not be Explained by the SM…….

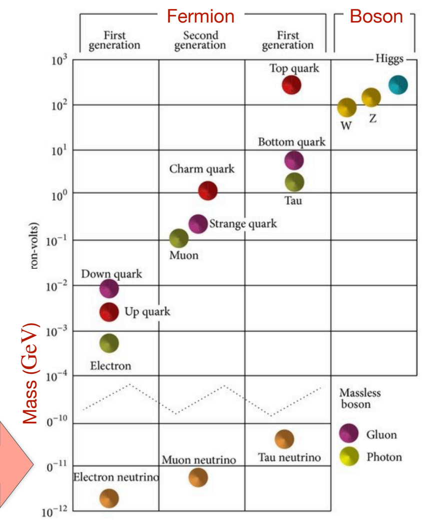
The identity of
Dark Matter



Matter-antimatter asymmetry of the Universe
Inflation
Dark energy



Why are there 3 generations ?
Why such significant mass hierarchies ?



Inclusion of gravity
Instability of the Higgs potential
Strong CP problem
….

Neutrino Mass$\neq$ 0
Mixing and CP violation

Point toward the existence of physics beyond the SM

To address the strong CP problem & dark matter and explain the Quark and lepton mass and mixing hierarchies including their CP violating phases,

we propose a Modular- and Gauge-invariant model within a 4D effective action,

based on $G_{\rm SM} \times U(1)_X \times SL(2,\mathbb{Z})$ symmetry.

★ String theory when compactified to 4-dim. can generically contain : Gauged $U(1)_X$

★ String Theory and the Emergence of the Fundamental Mathematical Structure : Modular Group $SL(2,Z)$

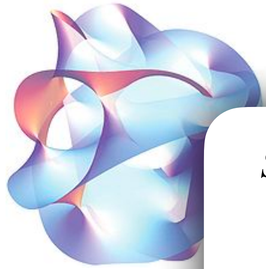
Contents

★ A 4D effective model with $G_{\rm SM} \times U(1)_X \times SL(2,\mathbb{Z})$ in string-derived supergravity

★ Anomaly cancellations & Flavor structure

★ The modulus VEV stabilizes near a fixed point, $\tau \approx i$

★ An example model & QCD axion mass



Gauged $U(1)_X$ and Modular Group $SL(2,Z)$

$SL(2,\mathbb{Z})$, especially *T*-duality, acts on the complex variable τ, varying in the upper-half complex plane $\mathrm{Im}(\tau) > 0$,

$$\tau \to \gamma\tau = \frac{a\tau + b}{c\tau + d} \quad (a, b, c, d \in Z,\ ad - bc = 1)$$

modular transformation

Fundamental Domain

$$\mathscr{D} \equiv \left\{\tau \in \mathscr{H} : -\frac{1}{2} \le \mathrm{Re}\,\tau < \frac{1}{2}, |\tau| > 1\right\} \cup \left\{\tau \in \mathscr{H} : -\frac{1}{2} < \mathrm{Re}\,\tau \le 0, |\tau| = 1\right\}$$

While no specific value of τ preserves the full modular symmetry, partial modular symmetries are retained at special symmetric points

$$\tau = i, i\infty, e^{i2\pi/3}$$

Ibanez & Lust 9202046
Novichkov, Penedo, Petcov, Titov 1811.04933
Gonzalo, Ibanez, Uranga 1812.06520

Under the modular transformation and the gauged $U(1)_X$,

the SUSY action $\mathcal{S} = \int d^4x d^2\theta d^2\bar{\theta}\, K(\Phi, \bar{\Phi}e^{2V}) + \left\{ \int d^4x d^2\theta \left(W(\Phi) + \frac{f_{ab}(\Phi)}{4} \mathscr{W}^{a\alpha}\mathscr{W}^b_\alpha \right) + \mathrm{h.c.} \right\}$

should be invariant

moduli

with $\Phi = (\tau, U_X, S, \ldots; \varphi, \varphi_X \ldots)$

matter superfields

Its invariance can strictly constrain the Kahler potential, Superpotential, and Gauge kinetic function

Anomalous triangle graphs associated with
- Modular invariance under the non-local modular group $SL(2,\mathbb{Z})$
- Gauged $U(1)_X$

Must Vanish within string-derived low energy theory

In the 4D $\mathcal{N} = 1$ string-derived supergravity framework $G(\Phi,\bar{\Phi}) = \dfrac{K(\Phi,\bar{\Phi})}{M_P^2} + \ln\dfrac{|W(\Phi)|^2}{M_P^6}$ & $f(\Phi)$

Kahler transformation

$$K(\Phi,\bar{\Phi}) \to K(\Phi,\bar{\Phi}) + \left(g(\Phi) + g(\bar{\Phi})\right)M_P^2$$
$$W(\Phi) \to W(\Phi)\, e^{-g(\Phi)}$$

$$K(\Phi,\bar{\Phi}) \to K(\Phi,\bar{\Phi}) + \left(g(\tau) + g(\bar{\tau})\right)M_P^2$$
$$W(\Phi) \to W(\Phi)\, e^{-g(\tau)}$$
$$f(\Phi)\,\mathscr{W}^\alpha\mathscr{W}_\alpha \to f(\Phi)\,\mathscr{W}^\alpha\mathscr{W}_\alpha$$

Under the modular transformation

When SM fermions transform nontrivially under $SL(2,\mathbb{Z})$, its invariance can offer a minimal flavor framework without excessive scalars

Binetruy & Dudas 9411413
Feruglio 1706.08749
Feruglio, Strumia, Titov 2305.08908
Kang & Ahn 2306.14467

- $SL(2,\mathbb{Z})$ mixed anomaly-free $\Rightarrow$ flavor structure
- This does not affect the strong CP phase
- In global SUSY limit $\Rightarrow$ its anomaly is not required to be free, but the strong CP phase remains unaffected
- Anomalies induced by the Kahler transformations match those from gaugino chiral rotations

When SM fermions transform nontrivially under $U(1)_X$
$\Rightarrow$ QCD axion to solve the strong CP problem
$\Rightarrow$ A dark matter candidate
$\Rightarrow$ Flavor structure

Ahn 1410.1634
Flavored PQ symmetry
Ahn 1604.01255
QCD axion as a bridge between string string theory and Flavor physics
Björkeroth, Chun, King 1806.00660
Flavorful axion phenomenology

Modular form

$U(1)_X$ charged scalar field

Yukawa couplings $\sim \quad Y_{ij}(\tau)\left(\dfrac{\Phi_X}{\Lambda}\right)^n = c_{ijX}\, F_{k_{ij}}(\tau)\left(\dfrac{\Phi_X}{\Lambda}\right)^n$

Polynomials in the Eisenstein series E_4 and E_6
Feruglio, Strumia, Titov 2305.08908

unit-magnitude Complex constants

that transform as $SL(2,\mathbb{Z})$ singlets, their holomorphicity across the fundamental domain (including $\tau = i\infty$) leads to nearly unique, weight-specific forms

Minimal set-up

Based on the 4D effective action derived from type-IIA intersecting D-brane (T-dual to magnetized D-brane models), we consider the following with minimal chiral superfields transforming under

$$G_{\rm SM} \times SL(2,\mathbb{Z}) \times U(1)_X$$

Axio-dilaton

$U(1)_X$-charged complex structure modulus

Kahler modulus

$$K = -M_P^2 \ln\left\{ \left(S + \bar{S} - h\,\tilde{c}\,\ln(-i\tau + i\bar{\tau})\right)\left(U_X + \bar{U}_X - \frac{\delta_X^{\rm GS}}{16\pi^2}V_X\right)^h \left(-i\tau + i\bar{\tau}\right)^h \right\}$$

with $h = 3$

$$+(-i\tau + i\bar{\tau})^{-k}|\varphi|^2 + Z_X\,\varphi_X^\dagger e^{-XV_X}\varphi_X + \ldots.$$

Matter field

$U(1)_X$-charged matter field

Vector superfield of $U(1)_X$ (including A_X^μ)

$$U_X = \frac{1}{g_X^2} + i\theta_X$$

$\delta_X^{\rm GS}$ characterizes the coupling of the anomalous gauge boson to the axion θ_X

$$W(\Phi) = \frac{1}{3}Y(\tau)\,\varphi_i\,\varphi_j\,\varphi_k + W(\tau, U_X, S)$$

Matter part

Moduli

$$W(S, U_X, \tau) = C_0\frac{e^{-\alpha S}M_P^3}{[\eta(\tau)]^{2h(1+\alpha\tilde{c})}} + \frac{M_P^3}{[\eta(\tau)]^{2h}}\left(A\,e^{-aU_X} - B\,e^{-bU_X}\right)$$

Explicitly break S-shift symmetry

$$f_{ab} = \delta_{ab}(S + U_X)$$

Type IIA

Under the modular transformation, invariance of the 4D action requires

$$\varphi_i \to (c\tau + d)^{-k_i}\varphi_i$$
$$S \to S - \tilde{c}\,\ln(c\tau + d)^h$$

★ $K(\Phi, \bar{\Phi}) \to K(\Phi, \bar{\Phi}) + \big(g(\tau) + g(\bar{\tau})\big)M_P^2$ $\quad g(\tau) = \ln(c\tau + d)^h$

This redundancy induces a modular anomaly in SUGRA

★ $f_{ab} \to f_{ab}^{1-\text{loop}} = \delta_{ab}(U_X + S - \tilde{c}\,\underbrace{\ln(c\tau + d)^h}_{g(\tau)})$

★ $W(\Phi) = \frac{1}{3} Y(\tau)\,\varphi_i\,\varphi_j\,\varphi_k + W(\tau, U_X, S)$

For $W(\Phi)$ to be modular invariant under the Kahler transformation ($W \to W e^{-g(\tau)}$),

$$Y(\tau) \to Y(\gamma\tau) = (c\tau + d)^{-k_Y}\,Y(\tau) \qquad \text{with } k_Y = h - (k_i + k_j + k_k)$$

The function $Y(\tau)$ can become independent of τ → **scalar potential**

Kang & Ahn 2306.14467

$$W_v = g_{\chi 0}\,\chi_0\,H_u H_d + \chi_0(g_\chi\,\chi\tilde{\chi} - \mu_\chi^2)$$

k_I	h	0	0	h	0	0
$U(1)_X$	0	0	0	0	$+1$	-1

ensures τ-independent modular form

$\chi(\tilde{\chi})$ charged by $+1(-1)$ are ensured by the extended $U(1)_X$
due to the holomorphy of the superpotential

dependent of τ → **Quark & Lepton Yukawa structure**

Since $\chi(\tilde{\chi})$ fields are SM gauge singlets with modular weight 0,
direct couplings to ordinary quarks and leptons are possible via Yukawa interactions

We assume :
symmetry-breaking scalars have modular weight zero

Otherwise, their VEVs must be zero

Their VEVs are invariant under the modular transformation

Under the modular transformation, matter fields transform in the form of chiral rotation

Canonically normalized fields $\hat{\varphi}$, defined by $\varphi_i = (K^{-1/2})_{ij}\,\hat{\varphi}_j$

$$\varphi_i \to (-i\tau + i\bar{\tau})^{k_i/2}\,\hat{\varphi}_i$$

$$\Downarrow$$

$$W = \frac{1}{3}\hat{Y}(\tau)\,\hat{\varphi}_i\,\hat{\varphi}_j\,\hat{\varphi}_k$$

with the normalized modular form in supergravity

$$\hat{Y}(\tau) = Y(\tau)\,e^{K/2M_P^2}(-i\tau + i\bar{\tau})^{(k_i+k_j+k_k)/2}$$

★ Under the modular transformations $\tau \to \gamma\tau \equiv \dfrac{a\tau + b}{c\tau + d}$ and $\varphi \to (c\tau + d)^{-k}\,\varphi$

the normalized matter fields and modular forms transform as

$$\hat{\varphi}_i \to \left(\frac{c\tau + d}{c\bar{\tau} + d}\right)^{-\frac{k_i}{2}}\hat{\varphi}_i\,, \qquad \hat{Y}(\tau) \to \left(\frac{c\tau + d}{c\bar{\tau} + d}\right)^{\frac{1}{2}(k_i+k_j+k_k-h)}\hat{Y}(\tau)$$

In the framework where the Einstein-Hilbert term is canonically normalized, the mass and kinetic terms for normalized chiral fermions $\hat{\psi}$ and gauginos $\hat{\lambda}$:

See also
Witten & Bagger PLB 115,202 (1982)
Wess & Bagger "Supersymmetry and supergravity"

$$-\frac{1}{2}\underbrace{e^{\frac{K}{2M_P^2}}(K^{-1/2})_i{}^k(K^{-1/2})_j{}^l(\mathscr{D}_k D_l W)}_{M_{\hat{\psi}}}\hat{\psi}^i\hat{\psi}^j + \frac{1}{2}\underbrace{\frac{1}{2}(\mathrm{Re}f)^{-1}_{ac}\,F^i\partial_i f_{cb}}_{M_{\hat{\lambda}}}\,\hat{\lambda}^a\hat{\lambda}^b + \mathrm{h.c.}$$

$$+\left(-\frac{i}{2}\bar{\hat{\psi}}^{\bar{i}}\bar{\sigma}^\mu\Gamma^i_{jk}\partial_\mu\phi^j\hat{\psi}^k + \mathrm{h.c.}\right) - i\bar{\hat{\psi}}^{\bar{i}}\bar{\sigma}^\mu D_\mu\hat{\psi}^j - i\bar{\hat{\lambda}}\bar{\sigma}^\mu D_\mu\hat{\lambda}$$

Spacetime covariant derivatives

Discrete-gauge symmetry

$$D_\mu \hat{\psi}^j = \partial_\mu \hat{\psi}^j + i q_K K_\mu \hat{\psi}^j \qquad (q_K = -1/2 \text{ for SM fermions})$$

$$D_\mu \hat{\lambda}^a = \partial_\mu \hat{\lambda}^a + f^{abc} A^b_\mu \hat{\lambda}^c + i q_K K_\mu \hat{\lambda}^a \qquad (q_K = +1/2 \text{ for gauginos})$$

Kahler connection

$$K_\mu = -\frac{i}{2M_P^2}(K_i \partial_\mu \phi^i - K_{\bar{i}} \partial_\mu \bar{\phi}^{\bar{i}})$$

In global SUSY limit $K_\mu \to 0$

Modular connection

$$\Gamma^i_{jk} = K^{i\bar{\ell}} \partial_j K_{k\bar{\ell}}$$

Under the Kahler transformation

$$K_\mu \to K_\mu - i\frac{h}{2}\left(\frac{c\partial\tau}{c\tau + d} - \frac{c\partial\bar{\tau}}{c\bar{\tau} + d}\right)$$

$$\Psi \to e^{-q_K \frac{g - \bar{g}}{2}} \Psi, \qquad \bar{\Psi} \to e^{-q_K \frac{g - \bar{g}}{2}} \bar{\Psi}$$

gaugino

SM fermion

Under the modular transformation, the term including modular connection, $\frac{1}{2}(\Gamma^{\varphi}_{\tau\varphi}\partial_\mu\tau - \Gamma^{\varphi}_{\bar{\tau}\varphi}\partial_\mu\bar{\tau})$, leads to

$$\frac{k_i}{2}\left(\frac{c\partial\tau}{c\tau + d} - \frac{c\partial\bar{\tau}}{c\bar{\tau} + d}\right) \cancel{- \frac{k_i}{2}\left(\frac{\partial_\mu\tau + \partial_\mu\bar{\tau}}{\tau - \bar{\tau}}\right)}$$

In the cusp limit $\tau \to i\infty$

$$\hat{\psi}_i \to e^{-\frac{k_i}{2}} \hat{\psi}, \qquad \bar{\hat{\psi}}_i \to e^{-\frac{k_i}{2}} \bar{\hat{\psi}}_i$$

$$\hat{\psi}_i \to \left(\frac{c\tau + d}{c\bar{\tau} + d}\right)^{k_{\hat{\psi}_i}} \hat{\psi}_i, \qquad \hat{\lambda} \to \left(\frac{c\tau + d}{c\bar{\tau} + d}\right)^{-\frac{h}{4}} \hat{\lambda} \qquad \text{with } k_{\hat{\psi}_i} = \frac{h}{4} - \frac{1}{2} k_i$$

Under the Kahler transformation and modular transformation, the Lagrangian is invariant (up to total derivative)

Induce chiral rotations in the fermionic path-integral measure, generating modular anomalies, similar to Adler-Bell-Jackiw anomaly

Quantum Modular anomaly appearing in the effective action

$$SL(2,\mathbb{Z}) \times \left\{ [U(1)_X]^2,\ [U(1)_Y]^2,\ [SU(2)_L]^2,\ [SU(3)_C]^2 \right\}$$

★ Anomaly generated by the chiral rotation of gauginos, exactly matches that from the Kahler transformation

$$-\frac{C}{8}\tilde{c}\left\{ -\big(g(\tau)+g(\bar\tau)\big)Q^{\mu\nu}Q_{\mu\nu} + i\big(g(\tau)-g(\bar\tau)\big)Q^{\mu\nu}\tilde{Q}_{\mu\nu} \right\}$$

$Q = G,\ W,\ Y,\ F_X$
for $SU(3)_C, SU(2)_L, U(1)_Y, U(1)_X$

In the action the gauge kinetic function is corrected to

$$f^{1-\text{loop}}_{ab}(\Phi) = \delta_{ab}(S+U_X) - \tilde{c}\,\underbrace{\ln(c\tau+d)^h}_{g(\tau)}$$

The modular anomaly generated by the chiral rotation of gauginos is cancelled

Derendinger, Ferrara, Kounnas, and Zwirner NPB 372,145(1992)
Kang & Ahn 2306.14467

★ The gaugino masses $M_{\hat\lambda} = \frac{1}{2}(\mathrm{Re} f)^{-1}_{ac} e^{\frac{K}{2M_P^2}} K^{i\bar j}\partial_i f_{cb}(D_{\bar j}\bar W)$ transform as $M_{\hat\lambda} \to \left(\frac{c\tau+d}{c\bar\tau+d}\right)^{\frac{h}{2}} M_{\hat\lambda}$

Gauginos do Not mix with SM fermions ⇒ their contribution to modular anomalies cancels

$$\arg(M_{\hat\lambda}) = 0$$

The gluino contribution to the strong CP phase vanishes

$$\vartheta_{\text{eff}} = \vartheta_{\text{QCD}} + \arg\left[\det(M_u M_d)\right] + A_C \arg\left(\frac{c\tau+d}{c\bar\tau+d}\right)$$

Anomaly generated in the effective action by the chiral rotation of SM fermions

Anomaly coefficient

$$SL(2,\mathbb{Z})\times[SU(3)_C]^2 \Rightarrow A_C=\sum_{i=1}^{3}\left(2k_{\hat{Q}_i}+k_{\hat{U}_i^c}+k_{\hat{D}_i^c}\right)$$

$$SL(2,\mathbb{Z})\times[U(1)_{\rm EM}]^2 \Rightarrow A_E=\frac{2}{3}\sum_{i=1}^{3}\left\{3(k_{\hat{L}_i}+k_{\hat{\ell}_i^c})+5k_{\hat{Q}_i}+4k_{\hat{U}_i^c}+k_{\hat{D}_i^c}\right\}$$

$$SL(2,\mathbb{Z})\times[SU(2)_L]^2 \Rightarrow A_L=\sum_{i=1}^{3}\left(k_{\hat{L}_i}+3k_{\hat{Q}_i}\right)$$

$$SL(2,\mathbb{Z})\times[U(1)_Y]^2 \Rightarrow A_Y=\frac{1}{3}\sum_{i=1}^{3}\left(k_{\hat{Q}_i}+8k_{\hat{U}_i^c}+2k_{\hat{D}_i^c}+3k_{\hat{L}_i}+6k_{\hat{\ell}_i^c}\right)$$

Due to the modular- and SM gauge-invariant structure of the superpotential with non-negative weight modular forms

$$\sum_{i=1}^{3}(k_{\hat{Q}_i}+k_{\hat{U}_i^c})=0 \qquad W\sim Y_{\mathbf{1}}^{(k_{\hat{u}_i})}U_i^cQ_iH_u \;\downarrow\; k_{\hat{u}_i}$$

on Dirac field

$$\sum_{i=1}^{3}(k_{\hat{Q}_i}+k_{\hat{D}_i^c})=0$$

$$\sum_{i=1}^{3}(k_{\hat{L}_i}+k_{\hat{\ell}_i^c})=0$$

for $k_{H_{u(d)}}=0$

$$A_C=0=A_E \qquad A_L=-A_Y$$

even in the global SUSY limit

If GS counter terms are introduced to cancel the anomalies A_L and A_Y, certain anomalies, that were already cancelled, are reintroduced

$$\delta^{\rm GS}_{SL(2,\mathbb{Z})}\propto A_Q$$

$A_L=-A_Y$ should be zero

$$\sum_{i=1}^{3}\left(k_{\hat{L}_i}+3k_{\hat{Q}_i}\right)=0$$

$$SL(2,\mathbb{Z})\times[U(1)_X]^2 \Rightarrow A_{SX}=\sum_{i=1}^{3}\left\{k_{\hat{Q}_i}\left(2X_{Q_i}^2-X_{U_i^c}^2-X_{D_i^c}^2\right)+k_{\hat{L}_i}\left(2X_{L_i}^2-X_{\ell_i^c}^2\right)+\underbrace{k_{\hat{N}_i^c}X_{N_i^c}^2}_{\text{Neutrino mass}}\right\}$$

Must Vanish

Constrain flavor-dependent $U(1)_X$ charges and the modular weights of right-handed neutrinos N_i^c

Gauged $U(1)_X$ Anomaly-free

The theory should also be invariant under the $U(1)_X$ gauge transformation $V_X \to V_X + i(\Lambda_X - \bar{\Lambda}_X)$

together with $\varphi_X \to e^{iX\Lambda_X}\varphi_X$ & $U_X \to U_X + i\dfrac{\delta_X^{\mathrm{GS}}}{16\pi^2}\Lambda_X$

$U_X = \dfrac{1}{g_X^2} + i\theta_X = \sigma + i\theta_X$

$\varphi_X|_{\theta=\bar{\theta}=0} = \dfrac{1}{\sqrt{2}}e^{i\frac{A_X}{f_X}}(f_X + h_X)$

$\xi_X = -\mathrm{Re}\Lambda_X|_{\theta=\bar{\theta}=0}$

So the axionic modulus θ_X and axion A_X have shift symmetries $\theta_X \to \theta_X - \dfrac{\delta_X^{\mathrm{GS}}}{16\pi^2}\xi_X$

$$A_X \to A_X + \frac{\delta_X^{\mathrm{GS}}}{\delta_X^{Q}} f_X\, \xi_X$$

with the $U(1)_X$ gauge field transformation $A_X^\mu \to A_X^\mu - \partial^\mu \xi_X$

$U(1)_X$ mixed anomaly coefficients

$U(1)_X \times [SU(3)_C]^2$

$U(1)_X \times [SU(2)_L]^2$

$U(1)_X \times [U(1)_Y]^2$

$U(1)_X \times [U(1)_X]^2$

The 4D effective action of the axions, θ_X and A_X, and its corresponding gauge field A_X^μ

$$K_{U_X\bar{U}_X}\left(\partial^\mu\theta_X - \frac{\delta_X^{\mathrm{GS}}}{16\pi^2}A_X^\mu\right)^2 + |D_\mu\varphi_X|^2 + D_X g_X X\left|\varphi_X\right|^2 - g_X\xi_X^{\mathrm{FI}}D_X$$

$$+\frac{1}{2}D_X^2 - \frac{1}{4g_X^2}F_X^{\mu\nu}F_{X\mu\nu} + \theta_X\frac{1}{2}\mathrm{Tr}(Q^{\mu\nu}\tilde{Q}_{\mu\nu}) + \frac{A_X}{f_X}\frac{\delta_X^Q}{32\pi^2}\mathrm{Tr}(Q^{\mu\nu}\tilde{Q}_{\mu\nu})$$

Non-trivial flux for the $U(1)_X$ gauge field living on the D6 branes

$D_X = g_X(\xi_X^{\mathrm{FI}} - X|\varphi_X|^2)$

String theory and particle physics : Ibanez & Uranga

hep-th 0309187 : Burgess, Kallosh, & Quevedo

$F_X^{\mu\nu} = \partial^\mu A_X^\nu - \partial^\nu A_X^\mu : U(1)_X$ gauge field strength

$|D_\mu\varphi_X|^2$ where the scalar fields φ_X couple to the $U(1)_X$ gauge boson

$D^\mu = \partial^\mu - iXA_X^\mu$ "spacetime covariant derivative involving $U(1)_X$ gauge boson"

Gauged $U(1)_X$ Anomaly-free

The $U(1)_X$ gauge invariance requires

★ $\delta_X^{\rm GS} = \alpha_X^Q\, \delta_X^Q$

$$\delta_X^G = 2{\rm Tr}[X_\psi T^2_{SU(3)}] = \sum_{i=1}^{3}\left(2X_{Q_i} + X_{D_i^c} + X_{U_i^c}\right)$$

$$\delta_X^W = 2{\rm Tr}[X_\psi T^2_{SU(2)}] = \sum_{i=1}^{3}\left(X_{L_i} + 3X_{Q_i}\right)$$

$$\delta_X^Y = 2{\rm Tr}[X_\psi Y^2] = \sum_{i=1}^{3}\left(\frac{1}{3}X_{Q_i} + \frac{8}{3}X_{U_i^c} + \frac{2}{3}X_{D_i^c} + X_{L_i} + 2X_{\ell_i^c}\right)$$

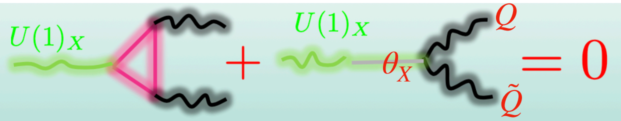
$$\delta_X^{F_X} = 2{\rm Tr}[X_\psi^3] = 2\sum_{i=1}^{3}\left(2X^3_{Q_i} + X^3_{D_i^c} + X^3_{U_i^c} + 2X^3_{L_i} + X^3_{\ell_i^c} + X^3_{N_i^c}\right)$$

In general

$\delta_X^Q \neq 0 \Rightarrow$ Anomalous $U(1)_X$!!

★ $\partial_\mu J^\mu_X = \dfrac{\delta_X^{\rm GS}}{32\pi^2}{\rm Tr}(Q^{\mu\nu}\tilde{Q}_{\mu\nu}) = -\partial_\mu J^\mu_\theta$

$J^\theta_\mu = K_{U_X\bar{U}_X}\dfrac{\delta_X^{\rm GS}}{8\pi^2}\partial_\mu\theta_X$ and $J^X_\mu = -\,iX\varphi_X^\dagger\overleftrightarrow{\partial}_\mu\varphi_X$



Anomaly cancellation !!

The anomaly generated by the triangle graph is cancelled by diagram in which the anomalous $U(1)_X$ mixes with the axionic modulus

The introduction of FI term $\mathscr{L}_X^{\rm FI} = \xi_X^{\rm FI} g_X\, D_X$ leads to the D-term potential for the anomalous $U(1)_X$

$$V_D = \frac{1}{U_X + \bar{U}_X}\left(-\xi_X^{\rm FI} + X\left|\varphi_X\right|^2\right)^2$$

$$\xi_X^{\rm FI} = hM_P^2\frac{\delta_X^{\rm GS}}{16\pi^2}\frac{\Delta\sigma}{\sigma_0}$$

Uplifting potential

$\Delta V \approx |V_{\rm AdS}|(\sigma_0/\sigma)^n$

Kachru, Kallosh, Linde, Maldacena, McAllister,Trivedi 0308055

★ Since the FI term is controlled by the string coupling, it can not be taken to be zero

But, the D-term can act as an uplifting potential, and then F-term must necessarily break SUSY

(for $X\,|\varphi_X|^2 = +\,|\varphi_X|^2 - |\tilde{\varphi}_X|^2$)

Burgess, Kallosh, Quevedo 0309187

★ The FI term can exceptionally vanish in flavored-axion framework when $\delta_X^{\rm GS} = 0$

Anomalous Global $U(1)_X$

The effective action is Canonically normalized by setting $\theta_X = a_\theta/8\pi^2 f_\theta$ with $f_\theta = \sqrt{2K_{U_X\bar{U}_X}/(8\pi^2)^2}$!!

The open string axion A_X (decay constant f_X) is mixed linearly with the closed string a_θ (decay constant f_θ)

$$\tilde{A} = \frac{A_X \frac{\delta_X^{\rm GS}}{2} f_\theta - a_\theta f_X}{\sqrt{f_X^2 + \left(\frac{\delta_X^{\rm GS}}{2} f_\theta\right)^2}} \approx A_X\,, \qquad G = \frac{a_\theta \frac{\delta_X^{\rm GS}}{2} f_\theta + A_X f_X}{\sqrt{f_X^2 + \left(\frac{\delta_X^{\rm GS}}{2} f_\theta\right)^2}} \approx a_\theta \quad \Longleftarrow \quad f_\theta \gg f_X$$

Since the $U(1)_X$ is gauged, G is eaten through the Higgs mechanism by the gauge boson making it massive

$\tilde{A}$ contributes to the QCD axion

★ The $U(1)_X$ gauge boson mass $m_X = \sqrt{2K_{U_X\bar{U}_X}(\delta_X^{\rm GS}/16\pi^2)^2 + 2f_X^2}$ is obtained by super-Higgs mechanism, while the D-term potential V_D remains

★ Below the scale m_X, the gauge boson decouples, leaving behind a low-energy symmetry which is anomalous global $U(1)_X$ → can be protected from quantum gravity effects

★ Non-perturbative quantum gravitational effects of the axionic shift symmetry by gravity can spoil the axion solution to the Strong CP problem

$$\partial_\mu J^\mu = \frac{\alpha_s}{8\pi} G^{a\mu\nu}\tilde{G}^a_{\mu\nu} + \Delta_{\rm UV}$$

$$\downarrow \qquad \downarrow$$

$$V(a) = V_{\rm QCD}(a) + \Delta V(a)$$

Kamionkowski, March-Russell 9202003
Kallosh, Linde, Linde, Susskind 9502069
Dvali 0507215
Ahn 1611.08359
Kang & Ahn 2306.14467

We assume:
$U(1)_X$-mixed gravitational anomaly-free by GS mechanism

Non-Anomalous Global $U(1)_X$

Flavor-dependent $U(1)_X$ charges satisfying $\boxed{\delta_X^Q = 0}$

$\Updownarrow$

$\boxed{\delta_X^{\mathrm{GS}} = 0}$ making $\partial_\mu J_X^\mu = 0$ and $\partial_\mu J_\theta^\mu = 0$

$\Rightarrow$

$\xi_X^{\mathrm{FI}} = 0$

D-flatness

★ The NG mode A_X is eaten by the $U(1)_X$ gauge boson A_X^μ, making it massive, no massless NG mode remains

★ At energies below the scale m_X, the $U(1)_X$ gauge boson decouples, leaving behind an anomaly-free global $U(1)_X$ symmetry

Reproduction $U(1)_{B-L}$

Assigning

$$X_{Q_i} = 1/3$$

$$X_{D_i^c} = X_{U_i^c} = -1/3$$

$$X_{L_i} = -1$$

$$X_{\ell_i^c} = 1$$

$$X_{N_i^c} = 1$$

$\Rightarrow$ $\delta_X^G = \delta_X^W = \delta_X^Y = \delta_X^{F_X} = 0$ $\Rightarrow$ Non-anomalous global $U(1)_X$ without massless NG mode!!

The moduli stabilization

A simple modular- and gauge-invariant superpotential

$$W(S, U_X, \tau) = C_0 \frac{e^{-\alpha S} M_P^3}{[\eta(\tau)]^{6(1+\alpha\tilde{c})}} + \frac{M_P^3}{[\eta(\tau)]^6}\left(Ae^{-aU_X} - Be^{-bU_X}\right)$$

Dedekind η-function

★ Under the modular transformation, $W \to We^{-g(\tau)}$ with $S \to S - \tilde{c}\,\ln(c\tau + d)^3$ & $\eta(\tau) \to (c\tau + d)^{1/2}\eta(\tau)$

★ Under the $U(1)_X$ gauge transformation, $A(\varphi_X/M_P) \to A(\varphi_X/M_P)e^{i\frac{a}{16\pi^2}\delta_X^{\rm GS}\Lambda_X}$ & $B(\varphi_X/M_P) \to B(\varphi_X/M_P)e^{i\frac{b}{16\pi^2}\delta_X^{\rm GS}\Lambda_X}$

The F-term potential $V_F = e^{K/M_P^2}\left\{K^{I\bar{J}}D_IW\bar{D}_{\bar{J}}\bar{W} - \frac{3}{M_P^2}|W|^2\right\}$

For small α

$$W = W(U_X, \tau) - \alpha\, C_0 \frac{M_P^3}{[\eta(\tau)]^6}\left(S + 6\tilde{c}\,\ln\eta(\tau)\right) + \frac{1}{2}\alpha^2 C_0 \frac{M_P^3}{[\eta(\tau)]^6}\left(S + 6\tilde{c}\,\ln\eta(\tau)\right)^2 - \frac{1}{6}\alpha^3 C_0 \frac{M_P^3}{[\eta(\tau)]^6}\left(S + 6\tilde{c}\,\ln\eta(\tau)\right)^3 + \ldots.$$

$$V_F = V_F^{(0)} + \alpha V_F^{(1)} + \alpha^2 V_F^{(2)} + \ldots$$

In the limit $\alpha \to 0$ (a racetrack type superpotential for U_X

the scalar potential $V_F^{(0)}$ has a local minimum at σ_0, τ_0

Kachru, Kallosh, Linde, Trivedi

hep-th: 0301240

Supersymmetric and Minkowski

$$W(\sigma_0, \tau_0) = 0 \qquad D_IW(\sigma_0, \tau_0) = 0 \qquad V(\sigma_0, \tau_0) = 0$$

$$V_F^{(0)}(\sigma_0) = 0$$

$$\partial V_F^{(0)}/\partial\sigma\,|_{\sigma=\sigma_0} = 0$$

$$C_0 = -A_0\left(\frac{aA_0}{bB_0}\right)^{-\frac{a}{a-b}} + B_0\left(\frac{aA_0}{bB_0}\right)^{-\frac{b}{a-b}} \qquad \sigma_0 = \frac{1}{a-b}\ln\left(\frac{aA_0}{bB_0}\right)$$

The VEV of the modulus τ

Solving $\partial V_F / \partial \tau = 0$

As a consequence of the supersymmetric & Minkowski solution, i.e. $C_0 = \ldots$ & $\sigma_0 = \ldots$ $\Rightarrow$ $V_F^{(0)}\Big|_{\sigma_0} = 0 = V_F^{(1)}\Big|_{\sigma_0}$

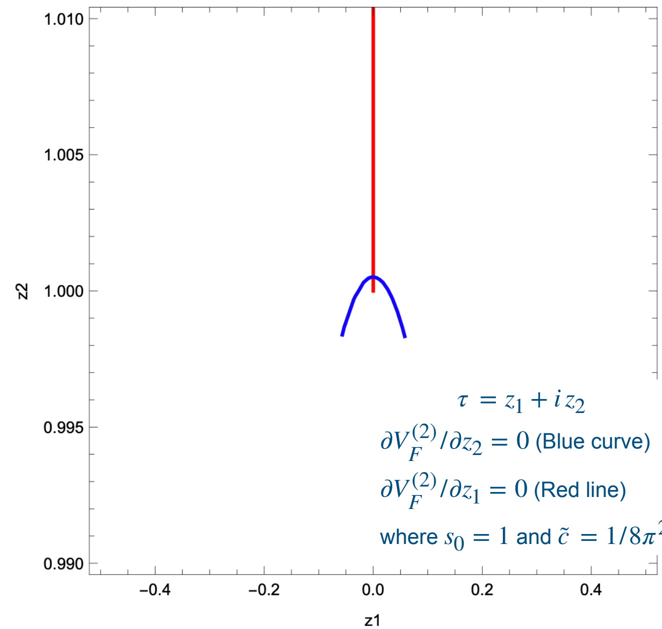
$$\frac{\partial V_F}{\partial \tau}\bigg|_{\tau=\tau_0(\alpha)} = \alpha^2 \frac{\partial V_F^{(2)}}{\partial \tau}\bigg|_{\tau_0} + \alpha^3 \left(\delta\tau \frac{\partial^2 V_F^{(2)}}{\partial \tau^2}\bigg|_{\tau_0} + \frac{\partial V_F^{(3)}}{\partial \tau}\bigg|_{\tau_0} \right) + \mathcal{O}(\alpha^4)$$

where $\tau_0(\alpha) \simeq \tau_0 + \alpha\,\delta\tau$

$$\boxed{\tau_0(\alpha) \approx i}$$

★ Although $SL(2,\mathbb{Z})$ — interpreted as T-duality — is treated as an exact discrete gauge symmetry, it becomes spontaneously broken when τ develops a VEV

★ At $\langle \tau \rangle \approx i$, no non-trivial subgroup survives at low-E



$$V_F^{(2)} = e^{K/M_P^2} \frac{M_P^6}{|\eta(\tau)|^{12}} |C_0|^2 \Big\{ |S + 6\tilde{c}\,\ln\eta(\tau)|^2 \left(K^{\tau\bar{\tau}} |H|^2 + \frac{1}{M_P^2} \right) - \frac{y}{M_P^2} \left(S + \bar{S} + 6\tilde{c}\,\ln \eta(\tau)\eta(\bar{\tau}) \right) + \frac{y^2}{M_P^2}$$
$$- 6\tilde{c} K^{\tau\bar{\tau}} \left((S + 6\tilde{c}\,\ln\eta(\tau)) H \frac{\eta'(\bar{\tau})}{\eta(\bar{\tau})} + h.c. \right) + 36\tilde{c}^2 K^{\tau\bar{\tau}} \left| \frac{\eta'(\tau)}{\eta(\tau)} \right|^2 \Big\}$$

SUSY breaking

$$y = S + \bar{S} - 3\tilde{c}\,\ln(-i\tau + i\bar{\tau})$$

For $\alpha = 0$ $\quad W(U_X, \tau) = \dfrac{M_P^3}{[\eta(\tau)]^{2h}}\left(C_0 + A\,e^{-aU_X} - Be^{-bU_X}\right)$

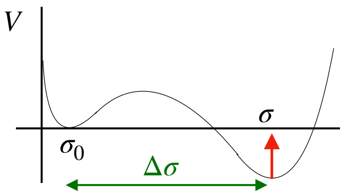
★ For $\alpha \neq 0$

SUSY is broken in the direction of S and τ

$$D_\tau W = -3W\left[\frac{1-\tilde{c}/y}{\tau - \bar{\tau}} + 2\frac{\eta'(\tau)}{\eta(\tau)}\right] - 6\tilde{c}\,\frac{\eta'(\tau)}{\eta(\tau)}\alpha C_0 \frac{e^{-\alpha S}M_P^3}{[\eta(\tau)]^{6(1+\alpha\tilde{c})}}$$

$$D_S W = -\frac{W}{y} - \alpha C_0 \frac{e^{-\alpha S}M_P^3}{[\eta(\tau)]^{6(1+\alpha\tilde{c})}} \quad , \qquad D_{U_X} W = 0$$

★



raises the AdS minimum to dS minimum

Kachru, Kallosh, Linde, Trivedi (0301240)
Kallosh, Linde (0411011)

In the U_X direction, a deeper AdS minimum emerges.
When a small weak-scale perturbation is introduced along U_X,
the potential minimum shifts from zero to a slightly negative value $V_{\text{AdS}} < 0$ at $(\sigma_0 + \delta\sigma, \tau_0, s_0)$.

AS in KL model, an uplift term (e.g., from a D-term) ΔV slightly shifts U_X, making $D_{U_X} W \neq 0$, but this contribution is strongly suppressed by m_σ

$$\langle D_\tau W\rangle \neq 0\,, \quad \langle D_S W\rangle \neq 0\,, \quad \langle D_{U_X} W\rangle \neq 0$$

$$\left|\frac{F^\tau}{F^S}\right| \approx 6\,\tilde{c}\,\frac{\langle K^{\tau\bar{\tau}}\rangle}{\langle K^{S\bar{S}}\rangle}\left|\frac{\eta'(\tau_0)}{\eta(\tau_0)}\right|\left|\frac{2s_0 - 3\tilde{c}\,\ln(2\mathrm{Im}\,\tau_0)}{-s_0 + 3\tilde{c}\big(2\ln\eta(\tau_0) + \ln(2\mathrm{Im}\,\tau_0)\big)}\right| \ll 1$$

SUSY is broken predominantly by the dilaton S and slightly by the modulus τ

This is decoupled from the AdS minimum of the U_X direction

When including the uplift term and $\alpha \neq 0$

$$m_{3/2}^2 = e^{\frac{K}{M_P^2}}\frac{|W|^2}{M_P^4} \simeq \frac{|V_F|}{3M_P^2} \approx \frac{|V_{\text{AdS}}| + \alpha^2|V_F^{(2)}|}{3M_P^2}$$

Modular- and Gauge-invariant Yukawa superpotential under $G_{\rm SM}\times SL(2,\mathbb{Z})\times U(1)_X$

Anomaly-cancellation conditions $A_C = 0 = A_E\,,\quad A_L = -A_Y = 0\,,\quad A_{SX} = 0$

	Left-handed Quarks			Right-handed Quarks					
Field	Q_1	Q_2	Q_3	d^c	s^c	b^c	u^c	c^c	t^c
k_I	$\frac{h}{2}+2$	$\frac{h}{2}+6$	$\frac{h}{2}+10$	$\frac{h}{2}-2$	$\frac{h}{2}-6$	$\frac{h}{2}-10$	$\frac{h}{2}-2$	$\frac{h}{2}-6$	$\frac{h}{2}-10$
$U(1)_X$	-20	-20	-8	40	30	12	43	30	8

$$
\begin{aligned}
W_q = &\Big[\alpha_t\, t^cQ_3 + \alpha_c\Big(\frac{\tilde\chi}{\Lambda}\Big)^{10} c^cQ_2 + \alpha_u\Big(\frac{\tilde\chi}{\Lambda}\Big)^{23} u^cQ_1 \\
&+\alpha_{t2}\Big(\frac{\chi}{\Lambda}\Big)^{12} Y_{\mathbf{1}}^{(4)} t^cQ_2 + \alpha_{t1}\Big(\frac{\chi}{\Lambda}\Big)^{12} Y_{\mathbf{1}}^{(12)} t^cQ_1 + \alpha_{c1}\Big(\frac{\tilde\chi}{\Lambda}\Big)^{10} Y_{\mathbf{1}}^{(4)} c^cQ_1\Big]H_u \\
&+\Big[\alpha_b\Big(\frac{\tilde\chi}{\Lambda}\Big)^{4} b^cQ_3 + \alpha_{b2}\Big(\frac{\chi}{\Lambda}\Big)^{8} Y_{\mathbf{1}}^{(4)} b^cQ_2 + \alpha_{b1}\Big(\frac{\chi}{\Lambda}\Big)^{8} Y_{\mathbf{1}}^{(8)} b^cQ_1 \\
&+\alpha_s\Big(\frac{\tilde\chi}{\Lambda}\Big)^{10} s^cQ_2 + \alpha_{s1}\Big(\frac{\tilde\chi}{\Lambda}\Big)^{10} Y_{\mathbf{1}}^{(4)} s^cQ_1 + \alpha_d\Big(\frac{\tilde\chi}{\Lambda}\Big)^{20} d^cQ_1\Big]H_d + \ldots
\end{aligned}
$$

$\chi\,(\tilde\chi)$: Modular weight 0

$+1(-1)$ under $U(1)_X$

$$\chi = \frac{v_\chi}{\sqrt{2}} e^{i\frac{A_X}{f_A}}\Big(1+\frac{h_\chi}{f_A}\Big) \qquad \tilde\chi = \frac{v_{\tilde\chi}}{\sqrt{2}} e^{-i\frac{A_X}{f_A}}\Big(1+\frac{h_{\tilde\chi}}{f_A}\Big)$$

$$f_A = \sqrt{v_\chi^2 + v_{\tilde\chi}^2}$$

★ All Yukawa coefficients α_i are unit-magnitude complex constants

Dots = infinite series of higher-dimensional operators induced by $\chi\tilde\chi$

$$1-\frac{\Delta_\chi^2}{1-\Delta_\chi^2} \le |\alpha_i| \le 1+\frac{\Delta_\chi^2}{1-\Delta_\chi^2} \quad \text{with } \Delta_\chi \equiv \frac{\langle\chi\rangle}{\Lambda}$$

★ $$\mathscr{L}_\vartheta = \Big(\vartheta_{\rm eff} + \frac{A_X}{F_a}\Big)\frac{g_s^2}{32\pi^2} G^{a\mu\nu}\tilde G^a_{\mu\nu}$$

with $F_a = \dfrac{f_A}{\delta_X^G}$

$$A_X = \langle A_X\rangle + a(x)$$

$U(1)_X\times[SU(3)_C]^2$ Color anomaly $\Rightarrow \delta_X^G = 67$

Modular- and Gauge-invariant Yukawa superpotential under $G_{\rm SM} \times SL(2,\mathbb{Z}) \times U(1)_X$

	Left-handed Leptons			Right-handed Leptons			Right-handed Neutrino		
Field	L_e	L_μ	L_τ	e^c	μ^c	τ^c	N_1^c	N_2^c	N_3^c
k_I	$\frac{h}{2}-18$	$\frac{h}{2}-22$	$\frac{h}{2}-14$	$\frac{h}{2}+18$	$\frac{h}{2}+22$	$\frac{h}{2}+14$	$\frac{h}{2}-6$	$\frac{h}{2}-6$	$\frac{h}{2}-6$
$U(1)_X$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}+23$	$-\frac{1}{2}+11$	$-\frac{1}{2}+7$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$

Charged-lepton

$$W_\ell = \Big[\alpha_\tau\Big(\frac{\tilde{\chi}}{\Lambda}\Big)^7\tau^c L_\tau + \alpha_\mu\Big(\frac{\tilde{\chi}}{\Lambda}\Big)^{11}\mu^c L_\mu + \alpha_e\Big(\frac{\tilde{\chi}}{\Lambda}\Big)^{23}e^c L_e$$
$$+\alpha_{\tau 2}\Big(\frac{\tilde{\chi}}{\Lambda}\Big)^7 Y_{\mathbf{1}}^{(8)}\tau^c L_\mu + \alpha_{\tau 1}\Big(\frac{\tilde{\chi}}{\Lambda}\Big)^7 Y_{\mathbf{1}}^{(4)}\tau^c L_e + \alpha_{e2}\Big(\frac{\tilde{\chi}}{\Lambda}\Big)^{23} Y_{\mathbf{1}}^{(4)} e^c L_\mu\Big] H_d + \ldots$$

Dirac Neutrino

$$W_\nu = \Big[\beta_{1e}Y_{\mathbf{1}}^{(24)}N_1^c L_e + \beta_{1\mu}Y_{\mathbf{1}}^{(28)}N_1^c L_\mu + \beta_{1\tau}Y_{\mathbf{1}}^{(20)}N_1^c L_\tau$$
$$+\beta_{2e}Y_{\mathbf{1}}^{(24)}N_2^c L_e + \beta_{2\mu}Y_{\mathbf{1}}^{(28)}N_2^c L_\mu + \beta_{2\tau}Y_{\mathbf{1}}^{(20)}N_2^c L_\tau$$
$$+\beta_{3e}Y_{\mathbf{1}}^{(24)}N_3^c L_e + \beta_{3\mu}Y_{\mathbf{1}}^{(28)}N_3^c L_\mu + \beta_{3\tau}Y_{\mathbf{1}}^{(20)}N_3^c L_\tau\Big] H_u + \ldots$$

Majorana Neutrino

$$+\frac{1}{2}\Big[\gamma_{11}Y_{\mathbf{1}}^{(12)}N_1^c N_1^c + \gamma_{12}Y_{\mathbf{1}}^{(12)}N_1^c N_2^c + \gamma_{13}Y_{\mathbf{1}}^{(12)}N_1^c N_3^c + \gamma_{22}Y_{\mathbf{1}}^{(12)}N_2^c N_2^c$$
$$+\gamma_{33}Y_{\mathbf{1}}^{(12)}N_3^c N_3^c + \gamma_{23}Y_{\mathbf{1}}^{(12)}N_2^c N_3^c\Big]\chi + \ldots$$

Seesaw mechanism

Minkowski PLB 67, 421(1977)
Yanagida (workshop 1979)
Gell-Mann, Ramond, Slansky, Nieuwenhuizen, Freeman (1979)

$U(1)_X \times [U(1)_{\rm EM}]^2$ EM anomaly $\Rightarrow E = \delta_X^W + \delta_X^Y = \dfrac{494}{3}$

Quark, lepton, and Flavored-QCD axion interactions

$$-\mathscr{L} \supset \overline{q_R^u}\,\mathscr{M}_u\,q_L^u + \overline{q_R^d}\,\mathscr{M}_d\,q_L^d + \frac{g}{\sqrt{2}}W_\mu^+\overline{q_L^u}\gamma^\mu\,q_L^d$$

$$+\overline{\ell_R}\,\mathscr{M}_\ell\,\ell_L + \frac{1}{2}\left(\overline{\nu_L^c}\quad\overline{N_R}\right)\begin{pmatrix} 0 & m_D^T \\ m_D & e^{i\frac{A_X}{f_A}}\,M_R \end{pmatrix}\begin{pmatrix}\nu_L \\ N_R^c\end{pmatrix} + \frac{g}{\sqrt{2}}W_\mu^-\overline{\ell_L}\gamma^\mu\,\nu_L + \text{h.c.}$$

Considering the canonically normalized fields $\varphi_i \to (-i\tau + i\bar{\tau})^{k_i/2}\,\hat{\varphi}_i$

the Yukawa coefficients transform as $\alpha_i \to (2\mathrm{Im}\,\tau)^{-k_i/2}\,\alpha_i$

$$\mathcal{M}_u = C_R^u \begin{pmatrix} \alpha_u\Delta_\chi^{23} & 0 & 0 \\ \alpha_{c1}(2\mathrm{Im}\,\tau)^{-2}\Delta_\chi^{10}\,Y_{\mathbf{1}}^{(4)} & \alpha_c\Delta_\chi^{10} & 0 \\ \alpha_{t1}(2\mathrm{Im}\,\tau)^{-4}\Delta_\chi^{12}\,Y_{\mathbf{1}}^{(8)} & \alpha_{t2}(2\mathrm{Im}\,\tau)^{-2}\Delta_\chi^{12}\,Y_{\mathbf{1}}^{(4)} & \alpha_t \end{pmatrix} C_L^u\,v_u \qquad C_R^u = \mathrm{diag}(e^{-i35\frac{A_X}{f_A}}, e^{-i22\frac{A_X}{f_A}}, 1), \qquad C_L^u = \mathrm{diag}(e^{i12\frac{A_X}{f_A}}, e^{i12\frac{A_X}{f_A}}, 1)$$

$$\Rightarrow \quad \hat{\mathscr{M}}_u = V_R^u\,\mathscr{M}_u\,V_L^{u\dagger} = \mathrm{diag}(m_u, m_c, m_t)$$

Unitarity mixing matrix up to $\mathcal{O}(\lambda^4)$

$$\mathcal{M}_d = C_R^d \begin{pmatrix} \alpha_d\,\Delta_\chi^{20} & 0 & 0 \\ \alpha_{s1}(2\mathrm{Im}\,\tau)^{-2}\Delta_\chi^{10}\,Y_{\mathbf{1}}^{(4)} & \alpha_s\,\Delta_\chi^{10} & 0 \\ \alpha_{b1}(2\mathrm{Im}\,\tau)^{-4}\Delta_\chi^{8}\,Y_{\mathbf{1}}^{(8)} & \alpha_{b2}(2\mathrm{Im}\,\tau)^{-2}\Delta_\chi^{8}\,Y_{\mathbf{1}}^{(4)} & \alpha_b\,\Delta_\chi^4 \end{pmatrix} C_L^d\,v_d \qquad C_R^d = \mathrm{diag}(e^{-i28\frac{A_X}{f_A}}, e^{-i18\frac{A_X}{f_A}}, 1), \qquad C_L^d = \mathrm{diag}(e^{i8\frac{A_X}{f_A}}, e^{i8\frac{A_X}{f_A}}, e^{-i4\frac{A_X}{f_A}})$$

$$\Rightarrow \quad \hat{\mathscr{M}}_d = V_R^d\,\mathscr{M}_d\,V_L^{d\dagger} = \mathrm{diag}(m_d, m_s, m_b)$$

★ $V_{\mathrm{CKM}} = V_L^u V_L^{d\dagger}$ $\qquad$ $\Delta_\chi = 0.634 \quad \Rightarrow \quad 0.33 \lesssim |\alpha_i| \lesssim 1.67$ and $\tan\beta \sim 6$

This can explain the current empirical data of quark mass & mixing

Quark, lepton, and Flavored-QCD axion interactions

$$-\mathscr{L} \supset \overline{q^u_R}\,\mathscr{M}_u\, q^u_L + \overline{q^d_R}\,\mathscr{M}_d\, q^d_L + \frac{g}{\sqrt{2}} W^+_\mu \overline{q^u_L}\gamma^\mu q^d_L$$

$$+\overline{\ell_R}\,\mathscr{M}_\ell\, \ell_L + \frac{1}{2}\left(\overline{\nu^c_L} \quad \overline{N_R}\right)\begin{pmatrix} 0 & m_D^T \\ m_D & e^{i\frac{A_X}{f_A}} M_R \end{pmatrix}\begin{pmatrix} \nu_L \\ N^c_R \end{pmatrix} + \frac{g}{\sqrt{2}} W^-_\mu \overline{\ell_L}\gamma^\mu \nu_L + \text{h.c.}$$

$$\mathcal{M}_\ell = C^\ell_R \begin{pmatrix} \alpha_e\,\Delta^{23}_\chi & \alpha_{e2} Y^{(4)}_{\mathbf{1}} \Delta^{23}_\chi (2\mathrm{Im}\,\tau)^{-2} & 0 \\ 0 & \alpha_\mu\,\Delta^{11}_\chi & 0 \\ \alpha_{\tau 1} Y^{(4)}_{\mathbf{1}} \Delta^{7}_\chi (2\mathrm{Im}\,\tau)^{-2} & \alpha_{\tau 2} Y^{(8)}_{\mathbf{1}} \Delta^{7}_\chi (2\mathrm{Im}\,\tau)^{-4} & \alpha_\tau\,\Delta^{7}_\chi \end{pmatrix} v_d \qquad C^\ell_R = \mathrm{diag}(e^{-i23\frac{A_X}{f_A}}, e^{-i11\frac{A_X}{f_A}}, e^{-i7\frac{A_X}{f_A}})$$

$$\Rightarrow \quad \hat{\mathscr{M}}_\ell = V^\ell_R\,\mathscr{M}_\ell\, V^{\ell\dagger}_L = \mathrm{diag}(m_e, m_\mu, m_\tau)$$

$$\theta^r_{23} \sim \mathcal{O}(\lambda^2)$$

These different corrections can explain

$$\Delta m^2_{\rm Atm} \simeq 2.5 \times 10^{-3}\,\mathrm{eV}^2$$

$$\Delta m^2_{\rm Sol} \simeq 7.4 \times 10^{-5}\,\mathrm{eV}^2$$

determine Normal mass ordering or Inverted mass ordering

$$M_R = (2\mathrm{Im}\,\tau)^{-6}\begin{pmatrix} \gamma_{11} Y^{(12)}_{\mathbf{1}} & \gamma_{12} Y^{(12)}_{\mathbf{1}} & \gamma_{13} Y^{(12)}_{\mathbf{1}} \\ \gamma_{12} Y^{(12)}_{\mathbf{1}} & \gamma_{22} Y^{(12)}_{\mathbf{1}} & \gamma_{23} Y^{(12)}_{\mathbf{1}} \\ \gamma_{13} Y^{(12)}_{\mathbf{1}} & \gamma_{23} Y^{(12)}_{\mathbf{1}} & \gamma_{33} Y^{(12)}_{\mathbf{1}} \end{pmatrix}\langle\chi\rangle$$

$$m_D = \begin{pmatrix} \beta_{1e} Y^{(24)}_{\mathbf{1}} (2\mathrm{Im}\,\tau)^{-12} & \beta_{1\mu} Y^{(28)}_{\mathbf{1}} (2\mathrm{Im}\,\tau)^{-14} & \beta_{1\tau} Y^{(20)}_{\mathbf{1}} (2\mathrm{Im}\,\tau)^{-10} \\ \beta_{2e} Y^{(24)}_{\mathbf{1}} (2\mathrm{Im}\,\tau)^{-12} & \beta_{2\mu} Y^{(28)}_{\mathbf{1}} (2\mathrm{Im}\,\tau)^{-14} & \beta_{2\tau} Y^{(20)}_{\mathbf{1}} (2\mathrm{Im}\,\tau)^{-10} \\ \beta_{3e} Y^{(24)}_{\mathbf{1}} (2\mathrm{Im}\,\tau)^{-12} & \beta_{3\mu} Y^{(28)}_{\mathbf{1}} (2\mathrm{Im}\,\tau)^{-14} & \beta_{3\tau} Y^{(20)}_{\mathbf{1}} (2\mathrm{Im}\,\tau)^{-10} \end{pmatrix} v_u$$

These corrections control the $U(1)_X$ breaking scale

$\Rightarrow$ Seesawing $\mathscr{M}_\nu \simeq -m_D^T M_R^{-1} m_D = U^*_\nu\,\mathrm{diag}\,.(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})\,U^\dagger_\nu$

★ $U_{\rm PMNS} = V^\ell_L\, U_\nu$

★ $\langle\chi\rangle = f_A/2 \sim 10^{12}\,\mathrm{GeV} \;\Rightarrow m_\nu \sim 0.05\,\mathrm{eV}$

This can explain the current empirical data of lepton mass & mixing

Quark, lepton, and Flavored-QCD axion interactions

Under the $U(1)_X$ transformation

$$\psi_f \to e^{iX_{\psi_f}\frac{\gamma_5}{2}\beta}\psi_f$$

Flavored-QCD axion interactions with Quarks (up to $\mathcal{O}(\lambda^4)$)

$$-\mathcal{L}^{aq} \simeq -\frac{\partial_\mu a}{2f_A}\Big\{23\,\bar{u}\gamma^\mu\gamma_5 u + 10\,\bar{c}\gamma^\mu\gamma_5 c + 20\,\bar{d}\gamma^\mu\gamma_5 d + 10\,\bar{s}\gamma^\mu\gamma_5 s + 4\,\bar{b}\gamma^\mu\gamma_5 b\Big\}$$

$$+\frac{\partial_\mu a}{2f_A}\Big\{\underline{12A_d\lambda_d^2 e^{i\varphi_d}\,\bar{s}\gamma^\mu(1-\gamma_5)b} + 12\lambda_d^3\left(B_d - A_d e^{i\varphi_d}\right)\bar{d}\gamma^\mu(1-\gamma_5)b + \mathrm{h.c.}\Big\} + \sum_{q=d,s,b,u,c,t}\left(m_q\bar{q}q - \bar{q}\,i\,\partial\!\!\!/\,q\right)$$

★ $B^\pm \to K^\pm + a$ ⇨ typically, $f_A \gtrsim 10^{5-6}\,\mathrm{GeV}$

★ No $s \to d + a$ (i.e. $K^+ \to \pi^+ + a$) up to $\mathcal{O}(\lambda^4)$

$V_R^{d(u)} = \mathbf{I} + \mathcal{O}(\lambda^4)$ and $C_L^d = \mathrm{diag}(e^{i8\frac{A_X}{f_A}}, e^{i8\frac{A_X}{f_A}}, e^{-i4\frac{A_X}{f_A}})$

CLEO
ARGUS
BaBar

Flavored-QCD axion interactions with Charged-leptons (up to $\mathcal{O}(\lambda^4)$)

$$-\mathcal{L}^{a\ell} \simeq -\frac{\partial_\mu a}{2f_A}\Big\{23\,\bar{e}\gamma^\mu\gamma_5 e + (11 - 4\theta_{23}^{r2})\,\bar{\mu}\gamma^\mu\gamma_5\mu + (7 + 4\theta_{23}^{r2})\,\bar{\tau}\gamma^\mu\gamma_5\tau\Big\}$$

$$-\frac{\partial_\mu a}{2f_A}\Big\{\underline{4\theta_{23}^r e^{i(\alpha_2^\ell - \alpha_3^\ell)}\,\bar{\mu}\gamma^\mu(1+\gamma_5)\tau} + \mathrm{h.c.}\Big\} + \sum_{\ell=e,\mu,\tau}\left(m_\ell\bar{\ell}\ell - \bar{\ell}\,i\,\partial\!\!\!/\,\ell\right)$$

★ $\tau \to \mu + a$ ⇨ $\theta_{23}^r \sim \mathcal{O}(\lambda^2)$ typically, $f_A \gtrsim 10^{5-6}\,\mathrm{GeV}$

★ No $\mu \to e + a$ up to $\mathcal{O}(\lambda^4)$

★ $|g_{aee}| = 23\frac{m_e}{f_A} < 4.3\times10^{-13}$ (95 % CL) RGB stars ⇔ $f_A \gtrsim 2.73\times10^{10}\,\mathrm{GeV}$

Bjorkeroth, Chun, King (1806.00660)
Calibbi, Goertz, Redigolo, Ziegler, Zupan (1612.08040)
Vega, Nath, Nellen, Peinado (2102.03631)

Quark-Lepton mass & CKM-PMNS

Using linear algebra tools : hep-ph 0501272

Antusch, Kersten, Lindner, Ratz, Schmidt

Taking in Quark sector

$\tau \approx i$

$\Delta_\chi = 0.634 \Rightarrow 0.33 \lesssim |\alpha_i| \lesssim 1.67$

$\tan\beta = 6.4$

$\theta^q_{23} = 2.323^\circ$

$\theta^q_{13} = 0.211^\circ$

$\theta^q_{12} = 13.017^\circ$

$\delta^q_{CP} = 63.436^\circ$

$m_d = 4.577$ MeV

$m_s = 103.800$ MeV

$m_b = 4.183$ GeV

$m_u = 2.488$ MeV

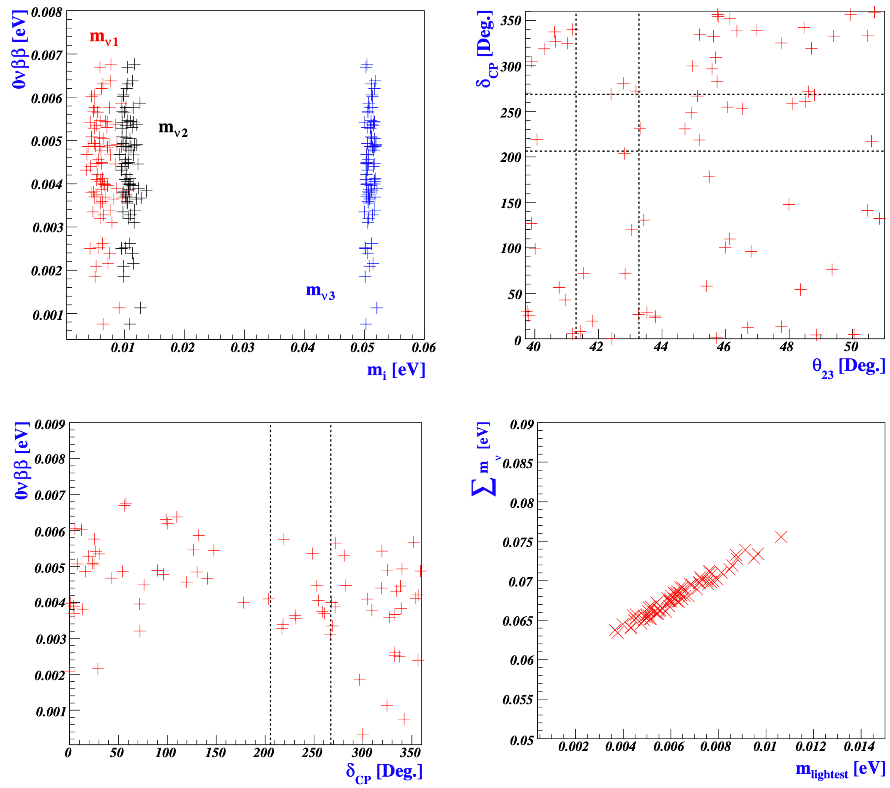
$m_c = 1.262$ GeV

$m_t = 173.582$ GeV

Taking in Lepton sector

$\langle \chi \rangle = 10^{12}$ GeV

Normal mass hierarchy



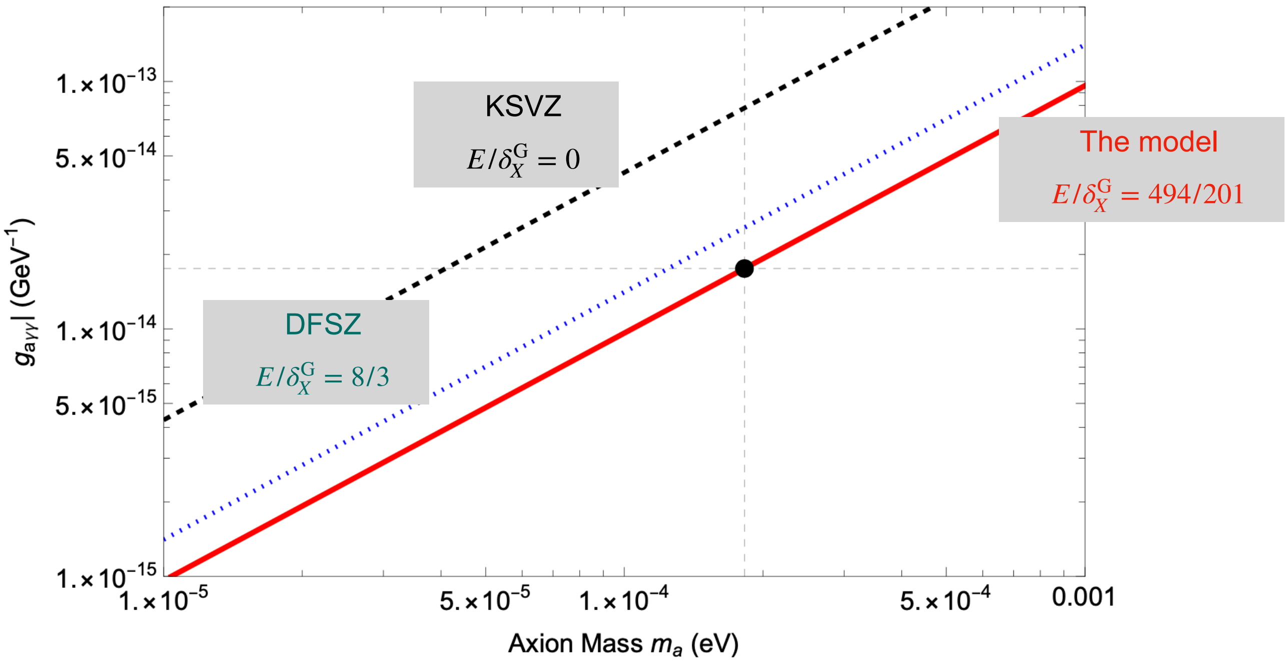
flavored-QCD axion

Taking in Neutrino sector

$$\langle\chi\rangle = 10^{12}\,\mathrm{GeV} \Leftrightarrow f_A = 2\times 10^{12}\,\mathrm{GeV}$$

$$m_a = 1.82\times 10^{-4}\,\mathrm{eV}$$

$$|g_{a\gamma\gamma}| = 1.75\times 10^{-14}\,\mathrm{GeV}^{-1}$$



Conclusion

We have constructed a 4D effective model within a string-derived supergravity framework under
$$G_{\rm SM} \times SL(2,\mathbb{Z}) \times U(1)_X$$

To simultaneously address the strong CP problem & Dark matter and account for the observed fermion mass and mixing hierarchies

We have shown that

★ $SL(2,\mathbb{Z})$- and $U(1)_X$ -mixed anomalies should vanish.

★ Anomalies induced by Kahler transformations match those generated by the chiral rotation of gauginos.

★ Despite the non-trivial transformation of SM fermions under $SL(2,\mathbb{Z})$, the strong CP phase remains unaffected, even in the global SUSY limit, where we assume the symmetry-breaking scalar fields have modular weights zero - so their VEVs stay invariant under the modular transformations.

★ The vanishing modular anomalies ($SL(2,\mathbb{Z}) \times [SU(3)_C]^2$ and $SL(2,\mathbb{Z}) \times [U(1)_{\rm EM}]^2$ guaranteed by modular- and SM gauge-invariance with non-negative weight modular forms) together with the anomaly-free conditions such as $SL(2,\mathbb{Z}) \times \left\{[SU(2)_L]^2, [U(1)_Y]^2, [U(1)_X]^2\right\}$ can determine: (i) flavor structure in both quark and lepton sectors, (ii) the effective strong CP phase, and (iii) potential scales for $U(1)_X$ breaking.

Conclusion

We have constructed a simple moduli superpotential to determine Yukawa couplings, gauge couplings, SUSY-breaking scale, and cosmological constant

- ★ We also demonstrated that the modulus VEV stabilizes near a fixed point (particularly $\tau \approx i$).
- ★ Although $SL(2,\mathbb{Z})$ -interpreted as T-duality- is treated as an exact discrete gauge symmetry, it becomes spontaneously broken when τ develops a VEV.
- ★ Notably, at $\langle\tau\rangle \approx i$, no non-trivial subgroup of the modular group survives at low energies.

The flavored $U(1)_X$ plays a crucial role in QCD axion solution to the strong CP problem, a candidate of DM, and fermion mass hierarchies

- ★ The anomaly coefficients, determined by the $U(1)_X$ charges of SM fermions, can either vanish or remain finite depending on the specific charge assignments across SM fermions.
- ★ For charge assignments $X_{Q_i} = \frac{1}{3}$, $X_{U_i^c} = X_{D_i^c} = -\frac{1}{3}$, $X_{L_i} = -1$, $X_{\ell_i^c} = 1$, and $X_{N_i^c} = 1$, this non-anomalous $U(1)_X$ reproduces $U(1)_{B-L}$ without leaving a NG mode.

A phenomenologically viable model has been presented

- ★ Consistent with the current experimental data on flavor mixing (PMNS and CKM) and quark and lepton mass spectra, including a normal hierarchical neutrino mass ordering.
- ★ Dominant flavored-QCD axion interactions to quarks and leptons : $B^{\pm} \to K^{\pm} + a$ and $\tau \to \mu + a$.
- ★ Predicting a QCD axion mass $m_a = 1.82 \times 10^{-4}$ eV and axion to photon coupling $|g_{a\gamma\gamma}| = 1.75 \times 10^{-14}$ GeV^{-1}.

Anomaly generated beyond that in the effective action

Furthermore, the chiral rotation of SM fermions (non-trivially charged under $SL(2,\mathbb{Z})$) can generate additional mixed anomalies beyond those from the gauge kinetic function

$$U(1)_{X(Y)} \times [SL(2,\mathbb{Z})]^2, \qquad U(1)_X \times U(1)_Y \times SL(2,\mathbb{Z})$$

do Not need to vanish, but should be cancelled either via GS mechanism or through appropriate fermion content.

We assume: canceled via GS mechanism mediated by moduli fields in the complete string compactification

cf.

In type II string vacuum, the anomalies should be cancelled by appropriate shifts of Ramond-Ramond axions in the bulk

Sagnotti 9210127

Ibanez, Rabadan, Uranga 9808139

Poppitz 9810010

Lalak, Lavignac, Nilles 9903160

Cvetic, Everett, Langacker, Wang 9903051