Probing the 3+1 model in the SHiP experiment

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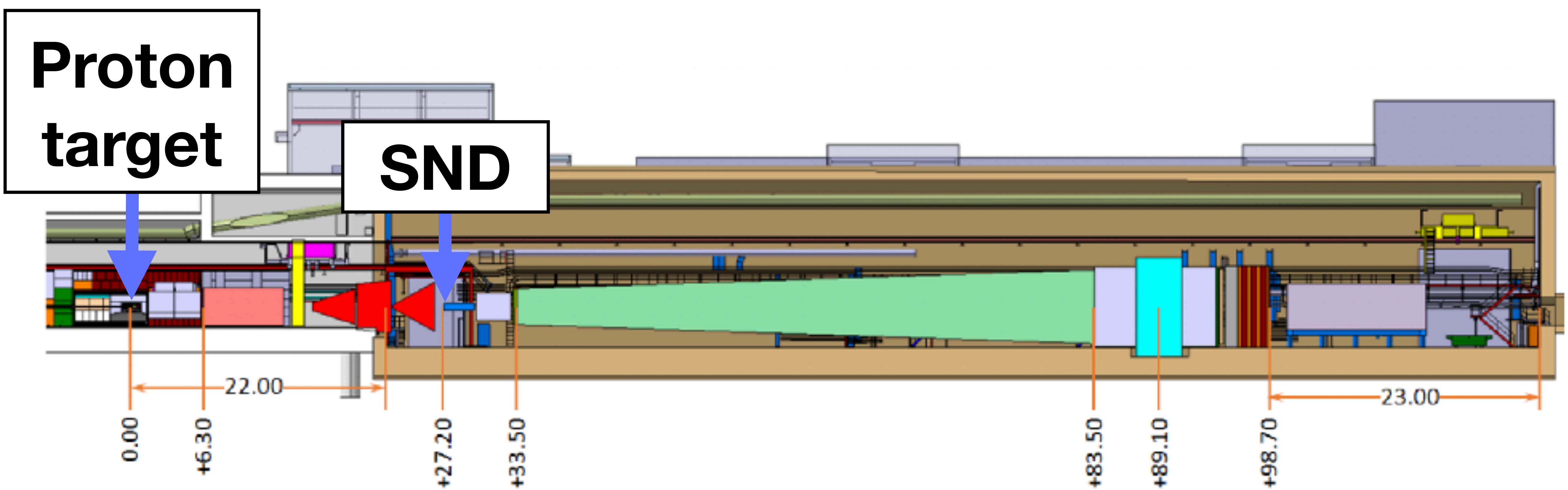
SHiP as the 3+1 model prober

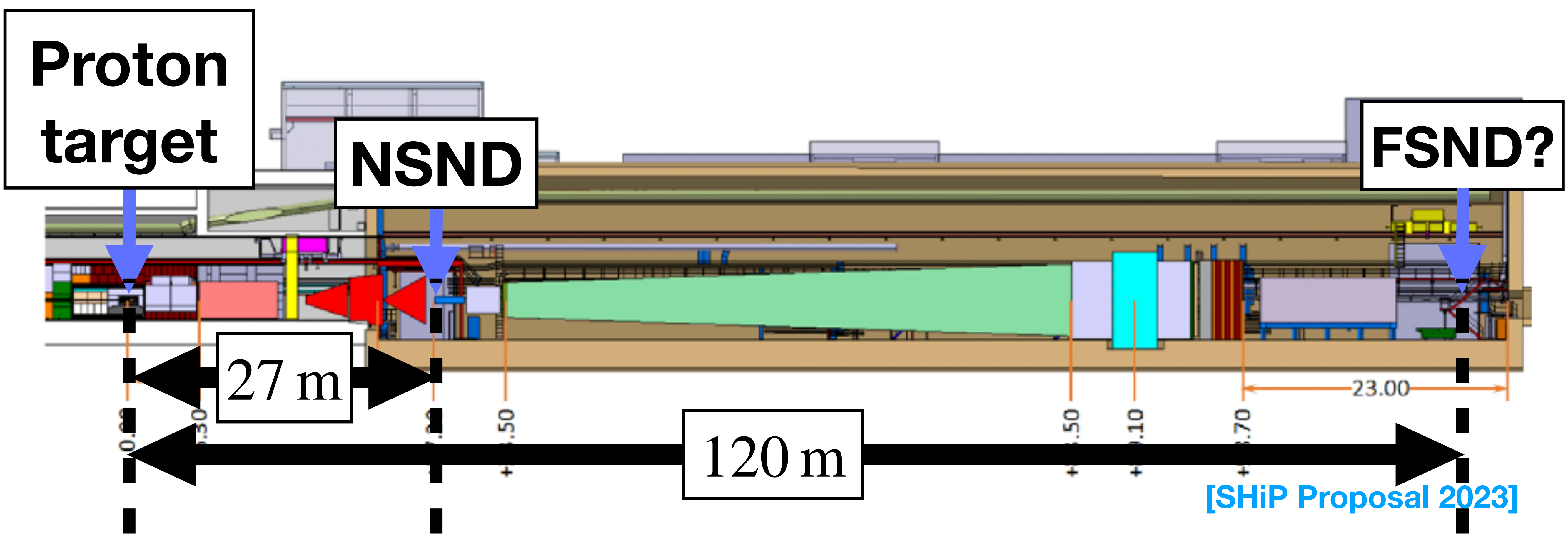
- Main objective: to utilize SHiP as a short baseline experiment
- The distance between Proton target and SND ⇒ baseline
- SND could probe CC events on every three flavor



[SHiP Proposal 2023]

SHiP as the 3+1 model prober

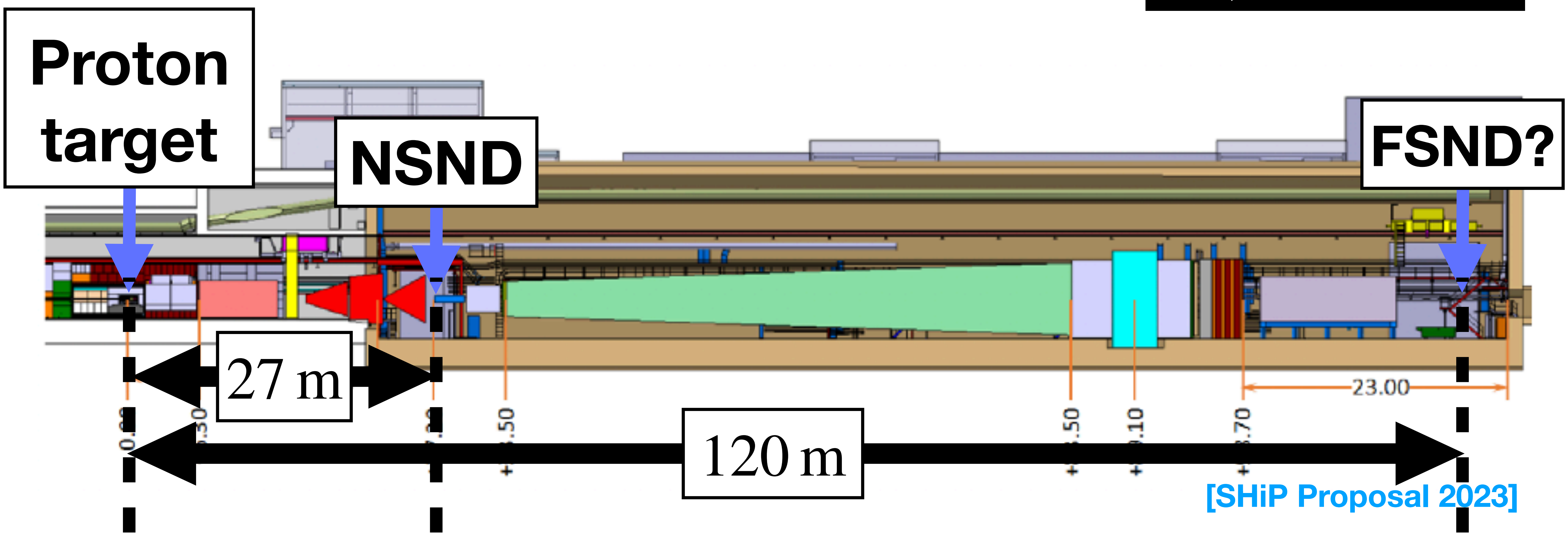
- Far SND (FSND) was proposed in our previous study to enhance sensitivity. 2403.04191
- For clarity, the original SND is labeled as Near SND (NSND).
- Length of SNDs: $2.6\,\mathrm{m}$



SHiP as the 3+1 model prober

- # of events at NSND: given by the proposal [SHiP Proposal 2023]
- We assumed that the energy spectrum is the same for both NSND and FSND, with a certain # of event ratio $\boxed{R_{\mathrm{F/N}}}$.

0 % , 10 % & 100 %



ν Oscillation in SHiP

Disappearance: $P_{\alpha\alpha} = 1 - 4|U_{\alpha 4}|^2(1 - |U_{\alpha 4}|^2)\sin^2 \dfrac{\Delta m_{41}^2 L}{4E_\nu}$

Appearance: $P_{\alpha\beta} = 4|U_{\alpha 4}|^2|U_{\beta 4}|^2\sin^2 \dfrac{\Delta m_{41}^2 L}{4E_\nu}$

Δm_{41}^2 with maximum sensitivity

NSND: $\dfrac{\Delta m_{41}^2 L}{4\left\langle E_\nu \right\rangle} \to \dfrac{\boxed{(10^3\,\mathrm{eV}^2)} \cdot (27\,\mathrm{m})}{4 \times (30\,\mathrm{GeV})} \approx 1$

FSND: $\dfrac{\Delta m_{41}^2 L}{4\left\langle E_\nu \right\rangle} \to \dfrac{\boxed{(250\,\mathrm{eV}^2)} \cdot (120\,\mathrm{m})}{4 \times (30\,\mathrm{GeV})} \approx 1$

The FC method with nuisance parameters

PhysRevD.57.3873

Sensitivity: confidence region of the most probable data set

The confidence region with given observations $\boldsymbol{D}$ & $\boldsymbol{\varphi}$:

$$\mathrm{CR}(\boldsymbol{D}, \boldsymbol{\varphi}; \alpha) = \{\boldsymbol{\theta} \,|\, \mathfrak{p}(\boldsymbol{\theta}; \boldsymbol{D}, \boldsymbol{\varphi}) > \alpha\},$$

where $\mathfrak{p}$ is the p-value which is defined as

$\alpha \equiv$ Significance Level
=1-(Confidence Level)

$$1 - \mathfrak{p}(\boldsymbol{\theta}; \boldsymbol{D}, \boldsymbol{\varphi}) \equiv \int_0^{\lambda(\boldsymbol{D}, \boldsymbol{\varphi}, \boldsymbol{\theta})} \rho(\lambda' \,|\, \boldsymbol{\theta}, \boldsymbol{\phi}) d\lambda',$$

where $\rho(\lambda' \,|\, \boldsymbol{\theta}, \boldsymbol{\phi})$ is a probability density function of the test statistic λ'.

The FC method with nuisance parameters

PhysRevD.57.3873

$$1 - \mathfrak{p}(\boldsymbol{\theta}; \boldsymbol{D}, \boldsymbol{\varphi}) \equiv \int_0^{\lambda(\boldsymbol{D},\boldsymbol{\varphi},\boldsymbol{\theta})} \rho(\lambda' | \boldsymbol{\theta}, \boldsymbol{\phi}) d\lambda'$$

Find $\rho(\lambda' | \boldsymbol{\theta}, \boldsymbol{\phi})$

$\Leftrightarrow$ Repeated sampling of $\boldsymbol{D}$ and $\boldsymbol{\varphi}$ assuming $\boldsymbol{\theta}$ and $\boldsymbol{\phi}$

$$\rho(\lambda' | \boldsymbol{\theta}, \boldsymbol{\phi}) = \int \delta[\lambda' - \lambda(\boldsymbol{O}, \boldsymbol{\varphi}', \boldsymbol{\theta})] \rho(\boldsymbol{O}, \boldsymbol{\varphi}' | \boldsymbol{\theta}, \boldsymbol{\phi}) d\boldsymbol{O}\, d\boldsymbol{\varphi}'$$

- $\boldsymbol{O}, \boldsymbol{\varphi}'$: samples of $\boldsymbol{D}, \boldsymbol{\varphi}$

Probability distribution of samples

$$P(\boldsymbol{O}, \boldsymbol{\varphi}' \,|\, \boldsymbol{\theta}, \boldsymbol{\phi}) = \prod_{\alpha,i} \mathrm{Pois}(O_{\alpha,i} \,|\, \mu_{\alpha,i}) \times \rho(\boldsymbol{\varphi}' \,|\, \boldsymbol{\phi}),$$

- $\boldsymbol{\phi}$ should be chosen for repeated sampling
- Profiled FC method: choose $\boldsymbol{\phi}$ that maximize $P(\boldsymbol{D}, \boldsymbol{\varphi} \,|\, \boldsymbol{\theta}, \boldsymbol{\phi})$.

  1503.07622, 2207.14353

  - $\mu_{\alpha,i} \equiv s_{\alpha,i} + b_{\alpha,i}$
  - The test statistic:

$$\lambda(\boldsymbol{O}, \boldsymbol{\varphi}', \boldsymbol{\theta}) \equiv \frac{\max_{\boldsymbol{\phi}} P(\boldsymbol{O}, \boldsymbol{\varphi}' \,|\, \boldsymbol{\theta}, \boldsymbol{\phi})}{\max_{\boldsymbol{\theta},\boldsymbol{\phi}} P(\boldsymbol{O}, \boldsymbol{\varphi}' \,|\, \boldsymbol{\theta}, \boldsymbol{\phi})}$$

Parametric bootstrap

https://hep-physics.rockefeller.edu/luc/talks/ParametricBootstrap.pdf

The point estimate (or mean) of nuisance parameters

$$\rho(\boldsymbol{\varphi} \,|\, \boldsymbol{\phi}) = \prod_j \mathcal{N}_{(-0.5,\infty)}(\varphi_j \,|\, \phi_j, \sigma_{\mathrm{norm}})$$

- The uncertainty of nuisance parameters is determined by the data of auxiliary measurements $\boldsymbol{D}_{\mathrm{aux}}$.
- Ideally, auxiliary measurements should be sampled as well as experimental data using.
- Instead, a *parametric bootstrap* samples the point estimate $\boldsymbol{\varphi}$ of nuisance parameters $\boldsymbol{\phi}$.

Signal estimate

$$s_{\alpha,i} = \sum_{\beta}^{e,\mu,\tau} (1 + \phi_\beta + \phi_{\beta,i}) \int_{E_{\mathrm{start},i}}^{E_{\mathrm{end},i}} dE_\nu \int_{L_0}^{L_0+L_\mathrm{d}} dL \int_0^\infty dE_{\mathrm{rec}} \varepsilon_\alpha P_{\beta\alpha} \frac{d^2 N_\beta}{dE_\nu dL} \frac{\sigma_{\nu_\alpha A}}{\sigma_{\nu_\beta A}} \rho(E_{\mathrm{rec}} \,|\, E_\nu)$$

- Energy reconstruction uncertainty: $20\ \%$ CERN-THESIS-2017-064
- Uncertainty of nuisance parameters ϕ: normalized as $\boxed{\sigma_{\mathrm{norm}}}$

$10\ \%\ \&\ 20\ \%$

- Detection efficiency:

$$\varepsilon_e = 30\ \%$$ 1904.05686

$$\varepsilon_\mu = 40\ \%$$ 1904.05686

$$\varepsilon_\tau = 10\ \%$$ CERN-THESIS-2021-091 $\phi_{\tau-h} \geq 2.8\ \mathrm{rad}$

- Nuisance parameters are the same for both NSND and FSND.

※ Any independency between NSND and FSND could lower the sensitivity.

Signal estimate

Nuisance parameters of flavor β

Contribution of flavor β on the 3+1 model

$$s_{\alpha,i} = \sum_{\beta}^{e,\mu,\tau} (1 + \boxed{\phi_\beta + \phi_{\beta,i}}) \boxed{\int_{E_{\mathrm{start},i}}^{E_{\mathrm{end},i}} dE_\nu \int_{L_0}^{L_0+L_\mathrm{d}} dL \int_0^\infty dE_\mathrm{rec} \varepsilon_\alpha P_{\beta\alpha} \frac{d^2N_\beta}{dE_\nu dL} \frac{\sigma_{\nu_\alpha A}}{\sigma_{\nu_\beta A}} \rho(E_\mathrm{rec} \,|\, E_\nu)}$$

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$\phi_{\tau-h} \geq 2.8\ \mathrm{rad}$

- Nuisance parameters are the same for both NSND and FSND.

※ Any independency between NSND and FSND could lower the sensitivity.

Background estimate

$$b_{\tau,i} = (1 + \phi_\mu + \phi_{\mu,i}) \int_{E_{\mathrm{start},i}}^{E_{\mathrm{end},i}} dE_\nu \int_{L_0}^{L_0+L_\mathrm{d}} dL \int_0^\infty dE_\mathrm{rec} R_{\mathrm{s/b}}^{-1} \varepsilon_\tau P_{\mu\mu} \frac{d^2 N_\tau}{dE_\nu dL} \rho(E_\mathrm{rec} \,|\, E_\nu)$$

- Background: tau only, signal-to-bkg ratio $R_{\mathrm{s/b}} = 2$ in the three-flavor model, assuming $\phi_{\tau-h} \geq 2.8\,\mathrm{rad}$. CERN-THESIS-2021-091

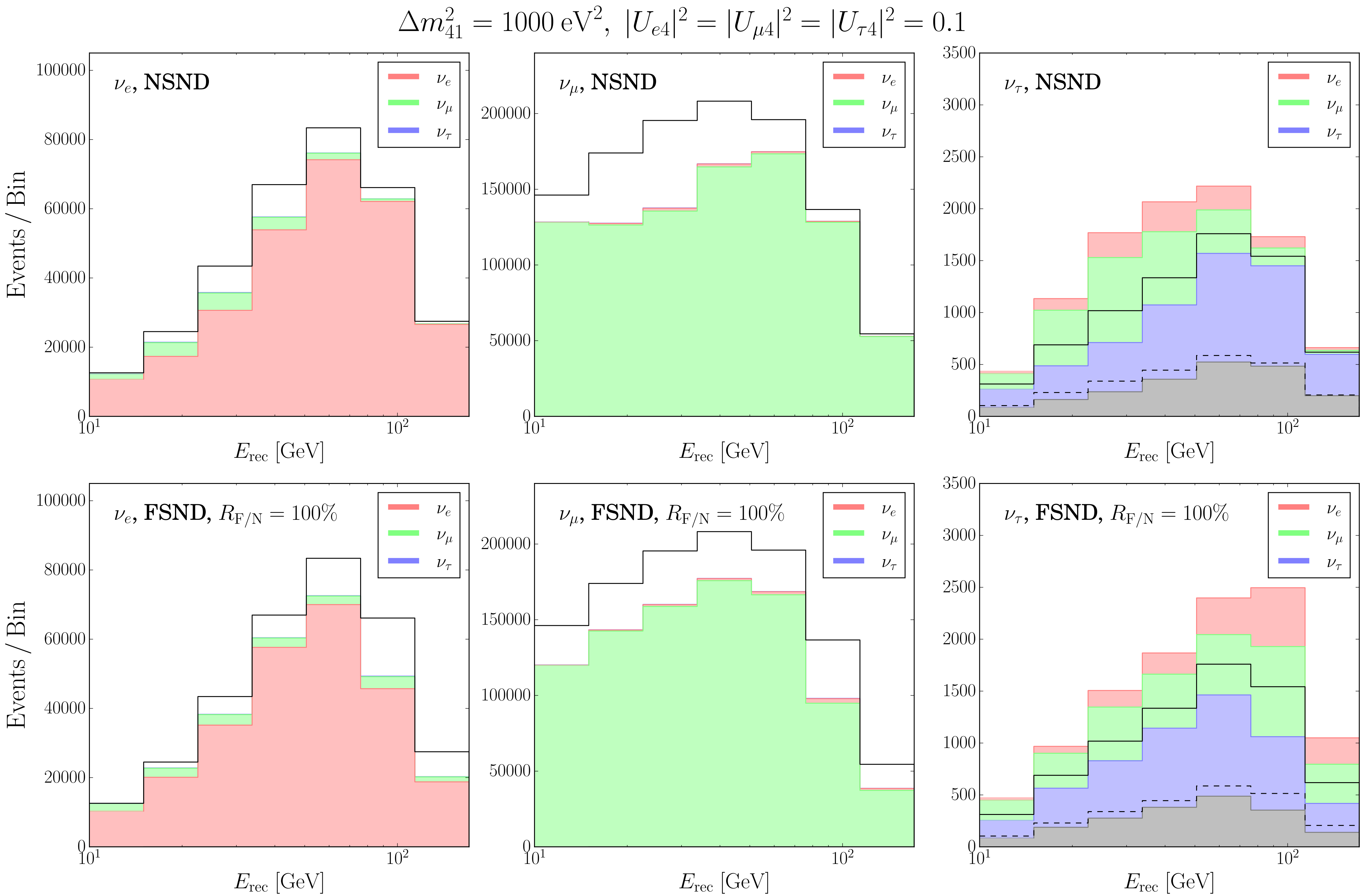
Purity of ν_e : $96\,\%$, ν_μ : $99.5\,\%$ at OPERA experiment

1904.05686

- The majority of the tau neutrino background originates from muon neutrinos.
- Therefore, the background is affected by muon neutrino disappearance.

Choice of energy bin

The energy bin range: $(E_i^{\rm start}, E_i^{\rm end})$, $E_0^{\rm start} = 10\,{\rm GeV}$, $E_i^{\rm end} = 1.5 \times E_i^{\rm start}$



CC DIS events at SND w/o mixing

Within 5 years of operation with a total of 2×10^{20} POT:

	Electron	Muon	Tau
Signal	3.2×10^5	1.1×10^6	7.2×10^4
Bkg	0	0	3.6×10^4

We draw the sensitivity for cases when

- $R_{\mathrm{F/N}} = 0 \Leftrightarrow$ NSND only (Blue)
- $R_{\mathrm{F/N}} = 10\ \%$ (Purple)
- $R_{\mathrm{F/N}} = 100\ \%$ (Red)

&

- $\sigma_{\mathrm{norm}} = 10\ \%$ (Dashed)
- $\sigma_{\mathrm{norm}} = 20\ \%$ (Solid).

Sensitivities of SHiP

Sensitivity: confidence region of the most probable dataset

The profiled FC method & a parametric bootstrap on ϕ are used.
PRD.57.3873, 1503.07622, 2207.14353
https://hep-physics.rockefeller.edu/luc/talks/ParametricBootstrap.pdf

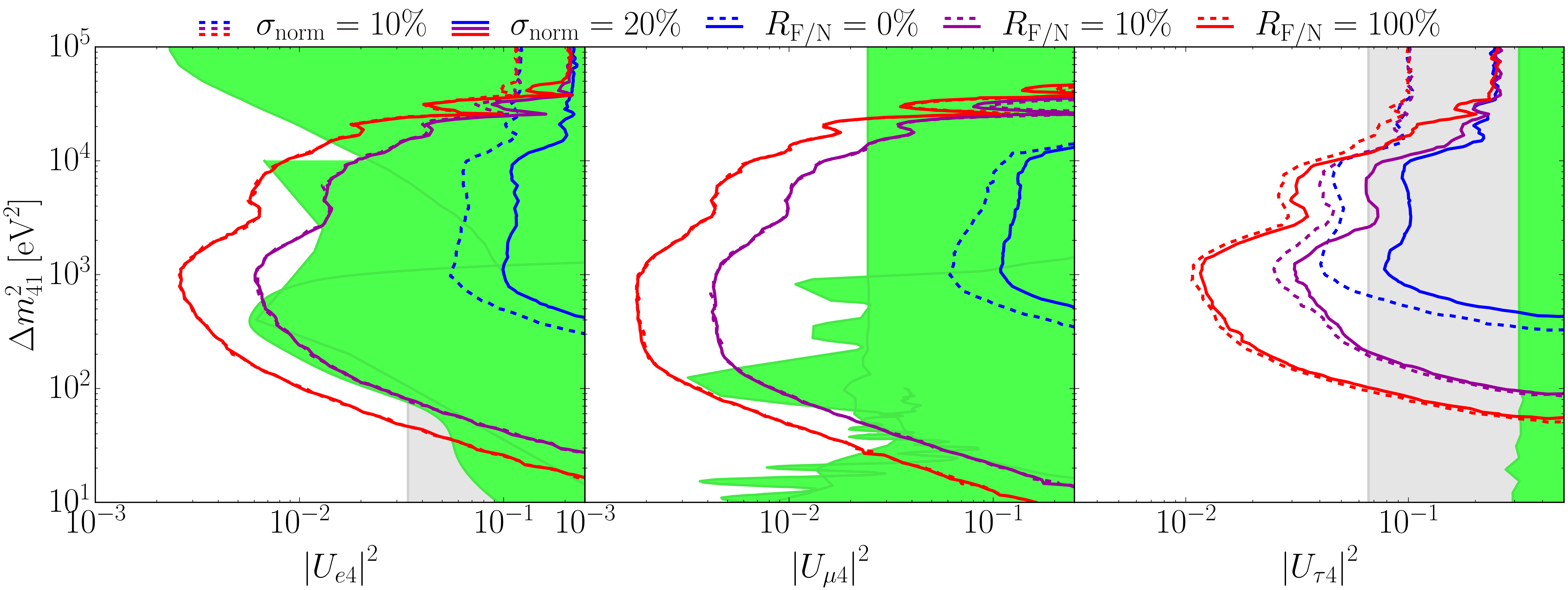
1. Sensitivities on the mixing with single flavor

- Only disappearance happens.
- The FC method on the plane $(\Delta m_{41}^2, |U_{\alpha 4}|^2)$.

2. Sensitivities on the mixing with two flavors

- The FC method on the plane $(|U_{\alpha 4}|^2, |U_{\beta 4}|^2)$, for given $\Delta m_{41}^2 = (50\,\mathrm{eV}^2,\ 250\,\mathrm{eV}^2,\ 1000\,\mathrm{eV}^2,\ 5000\,\mathrm{eV}^2)$.

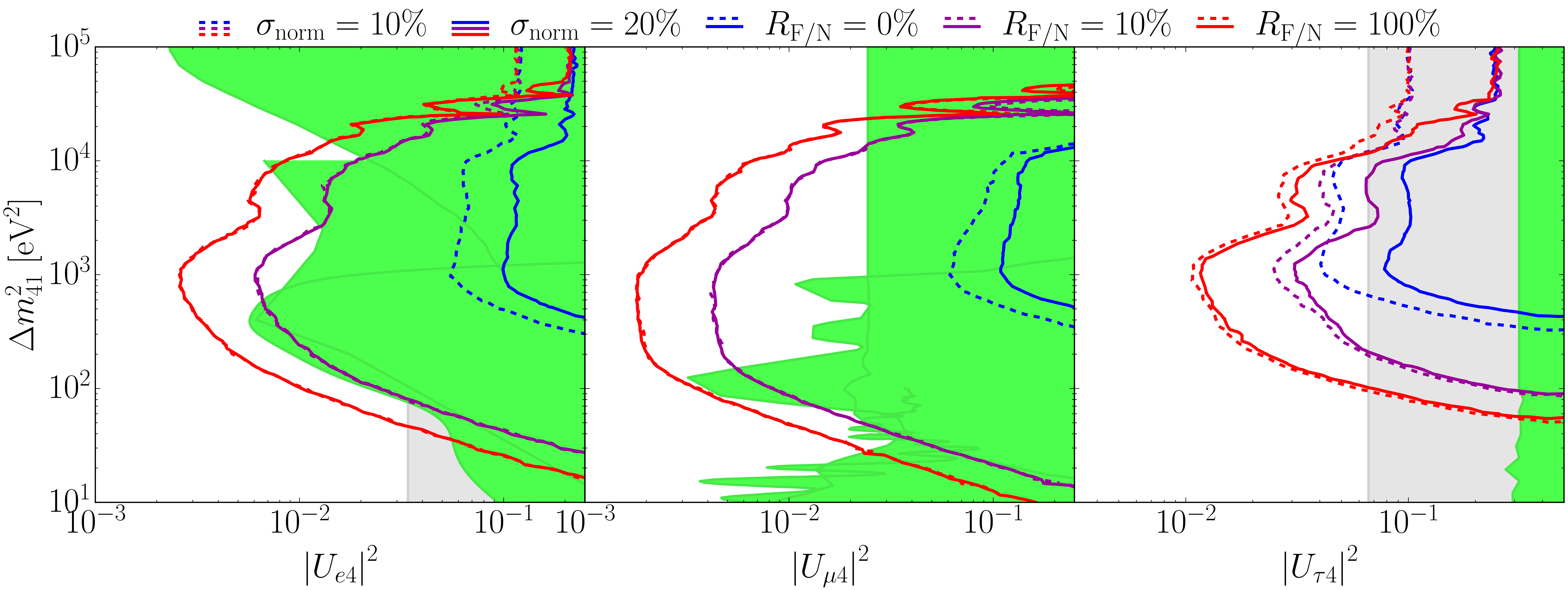
Sensitivities on single flavor mixing



Green: constraints excluded by other experiments

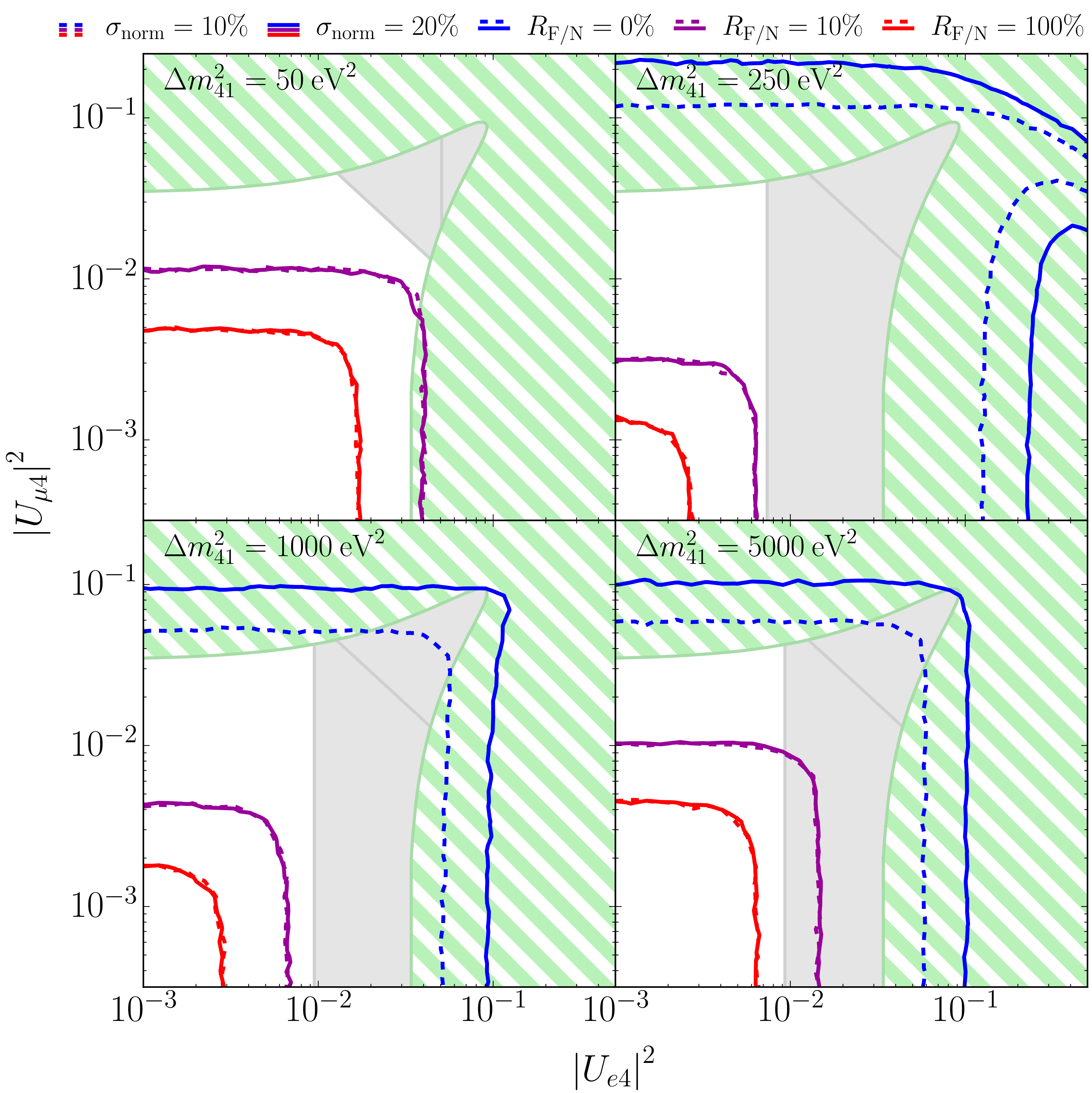
Grey: limit on mixing angle from different parameter space

Sensitivities on single flavor mixing



- Electron flavor: Tritium beta decay experiments
  1307.5687, 2201.11593, 2207.06337
- Muon flavor: CCFR, MiniBooNE+SciBooNE, MINOS
  PRL 52 (1984) 1384 1208.0322 1710.06488
- Tau flavor: NOvA, IceCube-DeepCore (Grey)
  2409.04553 2407.01314

Sensitivities on two flavor mixing ($e-\mu$)



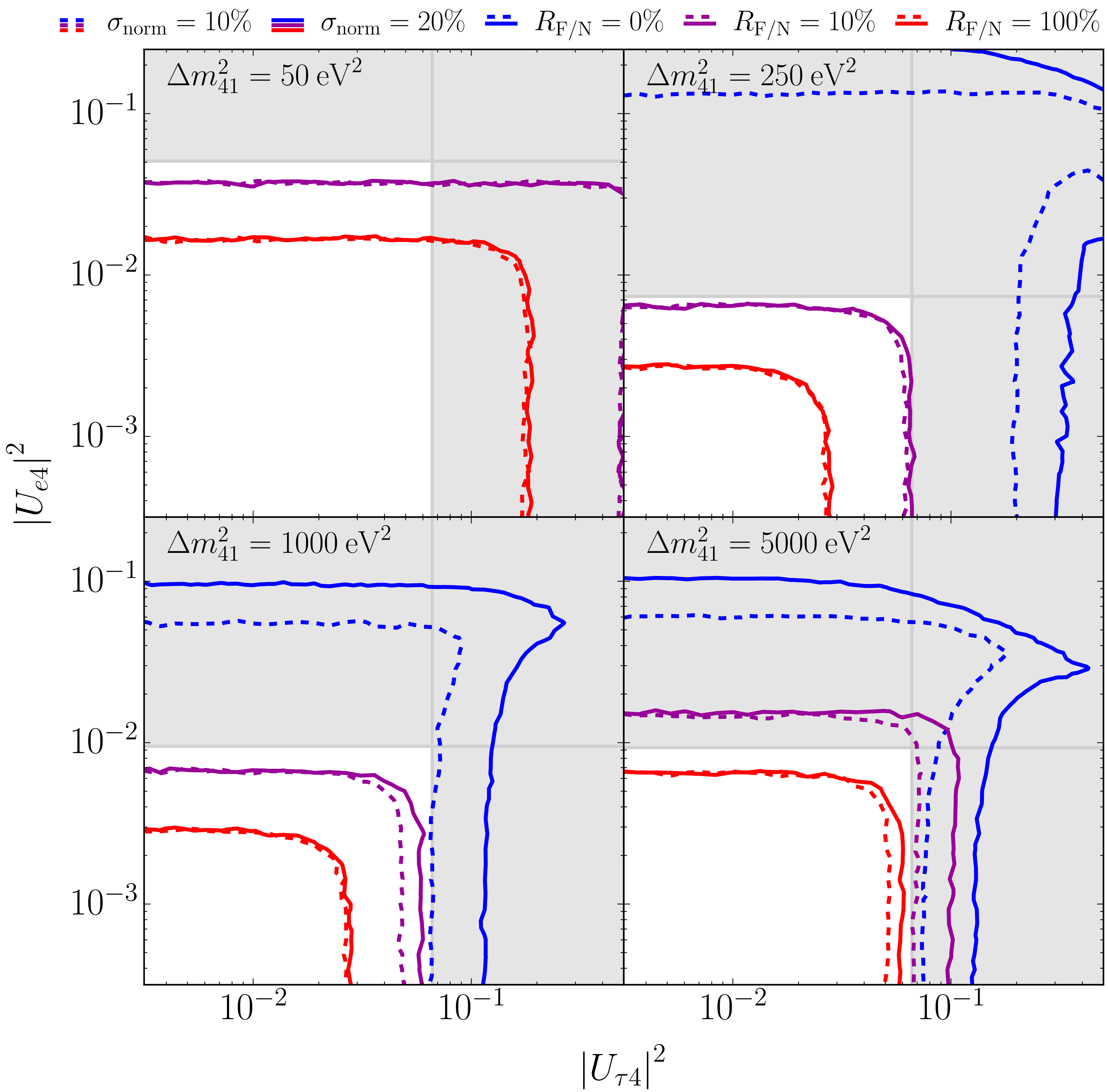
Dashed light green:

Super-Kamiokande
1410.2008

Grey:

Tritium beta decay exp.,
1307.5687, 2201.11593, 2207.06337
MicroBooNE (diagonal)
2210.10216

Sensitivities on two flavor mixing ($\tau - e$)



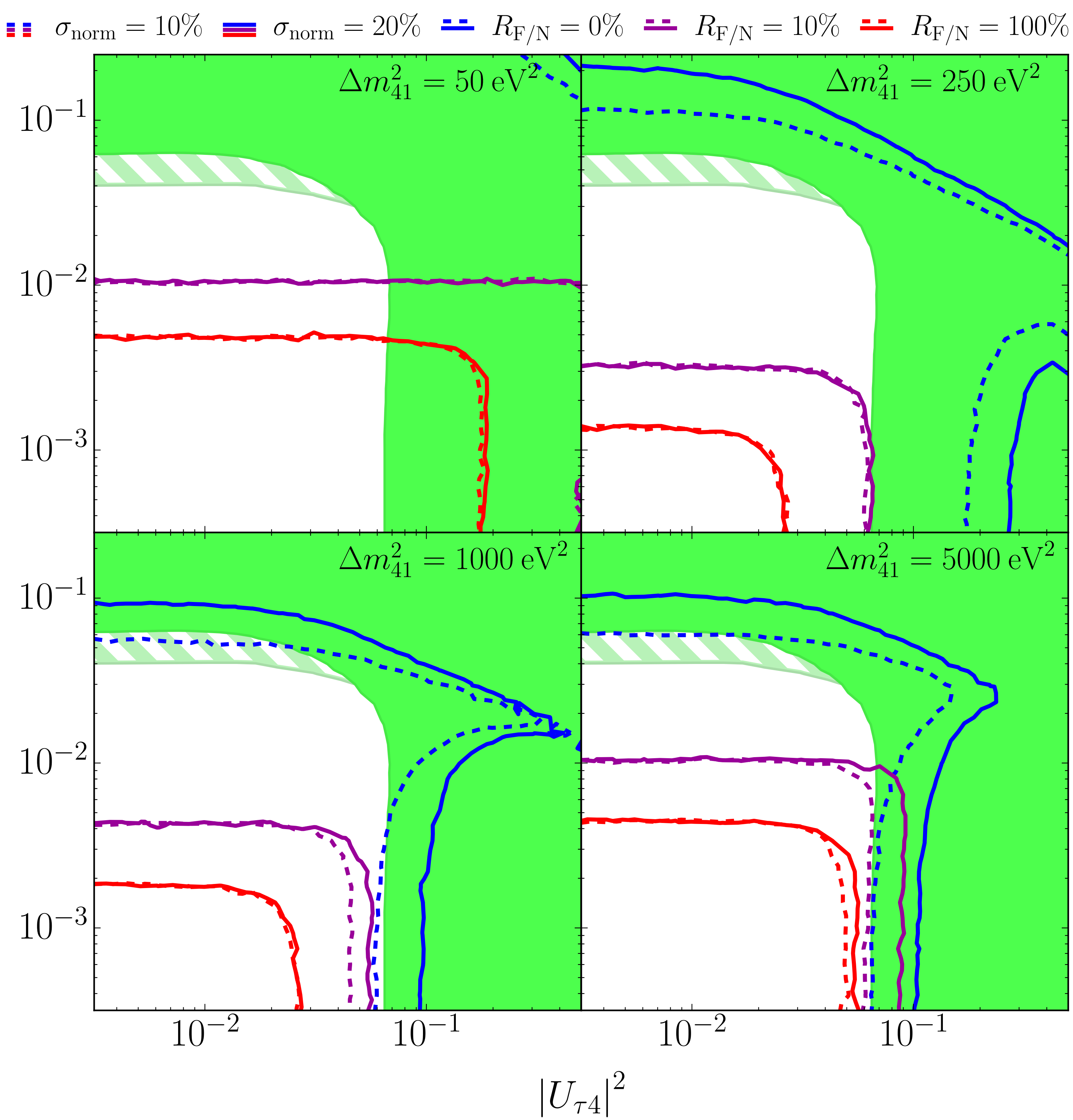
Horizontal grey:

Tritium beta decay exp., 1307.5687, 2201.11593, 2207.06337

Vertical grey:

IceCube-DeepCore 2407.01314

Sensitivities on two flavor mixing ($\tau - \mu$)



Green:

IceCube-DeepCore
2407.01314

Light Green:

Super-Kamiokande
1410.2008

Conclusion

- Using the FC method with a parametric bootstrap, we estimate sensitivity of SHiP to the $3+1$ model across six $(\sigma_{\text{norm}}, R_{\text{F/N}})$ configurations.
- Reducing σ_{norm} from 20% to 10% roughly doubles sensitivity in NSND, while dependency on σ_{norm} is disappeared with FSND.
- FSND with $R_{\text{F/N}} = 100\,\%$ enhances sensitivities on muon, tau, and electron mixing by factors of 10, 7, and 3 near $\Delta m_{41}^2 \sim 10^3\,\text{eV}^2$.
- The cancellation between appearance–disappearance generate kinks only on NSND only cases.