Cosmological Signatures of C-parity in SO(10)

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Outline

1. *Introduction*

Standard Model $(SU(3)_C \otimes SU(2)_L \otimes U(1)_Y)$

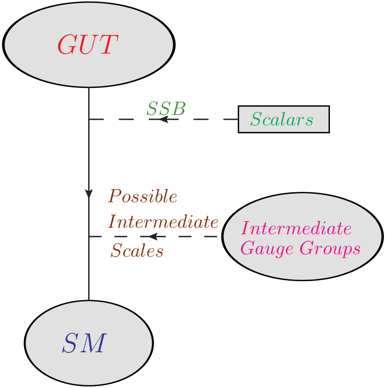
		Fields	Quantum numbers
Spin-$\frac{1}{2}$	Quarks	$Q_g^{i\alpha} = \{ \begin{pmatrix} u^\alpha \\ d^\alpha \end{pmatrix}_L , \begin{pmatrix} c^\alpha \\ s^\alpha \end{pmatrix}_L , \begin{pmatrix} t^\alpha \\ b^\alpha \end{pmatrix}_L \}$	$(3, 2, \frac{1}{6})$
		$u^C_{\alpha\,L},\ c^C_{\alpha\,L},\ t^C_{\alpha\,L}$	$(\overline{3}, 1, -\frac{2}{3})$
		$d^C_{\alpha L},\ s^C_{\alpha L},\ b^C_{\alpha L}$	$(\overline{3}, 1, \frac{1}{3})$
	Leptons	$\ell_g^i = \{ \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L , \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \}$	$(1, 2, -\frac{1}{2})$
		$e^C_L,\ \mu^C_L,\ \tau^C_L$	$(1, 1, 1)$
Spin-1	$SU(3)_C$	G^a_μ	$(8, 1, 0)$
	$SU(2)_L$	W^a_μ	$(1, 3, 0)$
	$U(1)_Y$	B_μ	$(1, 1, 0)$
Spin-0	Higgs	$\Phi^i = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	$(1, 2, \frac{1}{2})$

Table: *Fields in the Standard Model.*

Grand Unification Beyond the SM

- The basic idea in a Grand Unified Theory (GUT) is that the SM, $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, is embedded in a larger simple group, $\mathcal{G}$.

Schematic view



GUT Examples

- $SU(5)$ (rank $= 4$): $\bar{5} + 10 \Rightarrow$ SM fermions.

  Georgi, Glashow, PRL **32**, 438 (1974)

- $SO(10)$ (rank $= 5$): $16 \Rightarrow$ SM fermions $\oplus\ \nu_L^C$.

  Fritzsch, Minkowski, Ann. Phys. **93**, 93-266 (1975)

- $E(6)$ (rank $= 6$): $27 \Rightarrow$ SM fermions $\oplus\ \nu_L^C\ \oplus$

$$\underbrace{(2, \pm\frac{1}{2}, 1) + (1, -\frac{1}{3}, 3) + (1, \frac{1}{3}, \bar{3}) + (1, 0, 1)}_{\text{Exotic fermions}}.$$

  Gursey, Ramond, Sikivie, PLB **60** (1976) 177

  Shafi, PLB **79** (1978) 301

Discrete symmetries in $SO(10)$

- Two discrete Z_2 symmetries can appear on the breaking paths of $SO(10)$ to the SM:

1. Breaking of $SO(10)$ to the SM and subsequently to $SU(3)_c \times U(1)_{\text{em}}$ by scalars only in tensor reps. keeps a remnant Z_2 lying at its center.

2. C-parity (sometimes called as D-parity) appears at the left-right symmetric intermediate gauge symmetries depending on the VEV direction.

Kibble, Lazarides, Shafi, Phys. Lett. B **113** (1982)

Kibble, Lazarides, Shafi, PRD 26 (1982) 435

Chang, Mohapatra, Parida, PRL (1984) 1072

Lazarides, Shafi, JHEP 2019

2. *C-string, Monopole and Wall Bounded by Strings*

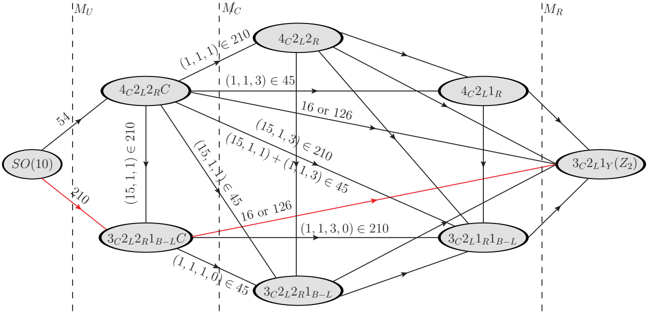
C-parity

- C interchanges left and right chiral fields accompanied with complex conjugation:

$$CT_L^3C^{-1} = -T_R^3,\ CT_R^3C^{-1} = -T_L^3,\ CT_{4c}^{\text{Cartan}}C^{-1} = -T_{4c}^{\text{Cartan}}.$$

- On the electric charge generator: $CQC^{-1} = -Q$.

Lazarides, Shafi, PLB 159 (1985) 261



Formation of C-strings and magnetic monopoles at GUT scale

$$SO(10) \xrightarrow{M_U} H = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times C/Z_2$$

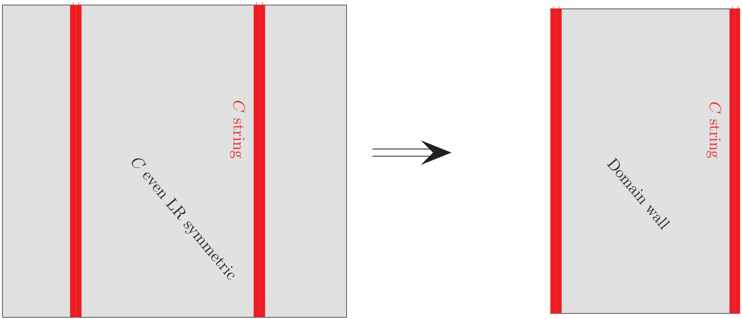
- Breaking of $SO(10)$ to a left-right symmetric model H that also leaves C unbroken yields a C-string at the GUT scale (string tension $\mu \sim M_U^2$.)
- This breaking $SO(10) \to H$ also produces a superheavy topologically stable magnetic monopole.

Lazarides, Magg, Shafi, PLB 97 (1980)

Walls bounded by strings

- C-string is not topologically stable because C-parity is spontaneously broken together with H.

- C-parity breaking at an intermediate scale ($\not{M}_C$) produces a domain wall, which connects two C-strings, dubbed as "wall bounded by string" (WBS). Kibble, Lazarides, Shafi, PRD 26 (1982) 435



Monopole problem

- Upper bound on comoving monopole number density: $Y_M = n_M/s \gtrsim 10^{-27}$. MACRO: EPJC 25 511, IceCube: PRL 128 (2022) 051101, ANTARES: JHEAp 34 (2022) 1, ...
- Topologically stable GUT monopoles must be diluted to be compatible with the observations.
- An inflation after GUT symmetry breaking helps. Guth 1980

Domain wall problem

- But wait! It inflates away the C-strings as well.
- C walls becomes effectively stable and dominate the energy budget of the universe after a time $(G\sigma)^{-1}$, where $\sigma \sim \cancel{M}_C{}^3$ is the wall tension. Zeldovich, Kobzarev, Okun, (1974)
- One way: Inflation happens after the formation of WBS and everything is inflated or symmetry was never restored. We do not have any observable signature of the topology of such symmetry breakings.

Partial experience of e-foldings and re-entering defects

- The GUT monopoles and the accompanied C-strings experience partial inflation.

- The strings and monopoles re-enter the horizon at a time t_F sec such that the monopole number density is sufficiently diluted:

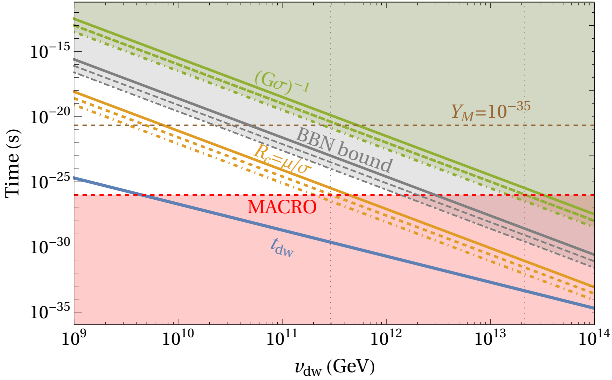
$$Y_M \simeq \frac{1}{V_h s(t_F)} \lesssim 10^{-27} \Rightarrow t_F \gtrsim 10^{-26} \text{ sec.}$$

- In addition, the strings should re-enter the horizon before the timescale $(G\sigma)^{-1}$ of the domain wall domination.

RM, Shafi, PhysRevD.111.075027

Re-entering defects and gravitational waves (GWs)

- $(G\sigma)^{-1} \gtrsim t_F \gtrsim 10^{-26}$ sec provides a window for the horizon re-entry of GUT monopoles and the C-strings.
- As strings reenter the horizon we get walls bounded by strings within the particle horizon, which eventually decay into GWs.



RM, Shafi, PhysRevD.111.075027

3 *Gravitational Waves from Walls Bounded by Strings*

Timescale for string network

- For a wall bounded by a string of radius of curvature R, the force per unit length on the string boundary $\sim \mu/R$ dominates over the wall tension σ for $R < \boxed{R_c = \mu/\sigma}$.
- $t_* = R_c$ to be the maximum timescale for domination by the string dynamics.
- The cosmic strings inter-commute, form loops before t_* with $R < t_*$ and can produce gravitational waves.

$$\boxed{f = \frac{2n}{l(t)} \frac{a(t)}{a_0}}$$

- The domain wall dynamics become dominant for $t > t_*$, and the string-wall networks collapse as the walls pull the strings.

Vilenkin, Everett, PRL **48** (1982) 1867

Martin, Vilenkin, PRL **77** (1996) 2879

Dunsky et. al., PRD **106** (2022) 075030

String dynamics domination before timescale R_c

The gravitational wave background from string loops:

$$\Omega_{\rm GW}(f) = \sum_{k=1}^{\infty} \Omega_{\rm GW}^{(k)}(f), \quad k \in Z^+,$$

$$\Omega_{\rm GW}^{(k)}(f) = \frac{1}{\rho_{c,0}} \int_{t_s}^{t_0} d\tilde{t} \left(\frac{a(\tilde{t})}{a(t_0)}\right)^5 \frac{\mathcal{F} C_{\rm eff}(t_i)}{(\Gamma G\mu + \alpha)\alpha t_i^4} \left(\frac{a(t_i)}{a(\tilde{t})}\right)^3 \frac{\Gamma k^{-4/3}}{\zeta(4/3)} G\mu^2 \frac{2k}{f} \Theta(R_c - t_i). \quad (1)$$

$\mathcal{F} \simeq 0.1$, $\Gamma \simeq 50$, $\alpha \simeq 0.1$ for string network in the scaling regime and $C_{\rm eff} = 5.7$ in the radiation dominated era. Vachaspati, Vilenkin, PRD 1985; Kibble, NPB 1985; Vilenkin, Shellard, CUP 2000; Damour, Vilenkin, PRD, 2001;Vanchurin, Olum, Vilenkin, PRD 2006; Ringeval, Sakellariadou, Bouchet, JCAP 2007; . Olmez, Mandic, Siemens, PRD 2010; Blanco-Pillado, Olum, Shlaer, PRD 2014; Blanco-Pillado, Olum, 2017;...

Wall dynamics domination after timescale R_c

- The WBS structures oscillate with constant size before collapsing at $t_d \sim 1/(G\sigma)$. Dunsky et. al., PRD 2022, Hiramatsu et. al., JCAP (2014)
- The gravitational wave background at t_d from oscillating WBS:

$$\Omega_{\rm GW}^{\rm osc}(\tilde{f}, t_d) = \frac{\sigma w}{m_{\rm Pl}^2}(\tilde{f} w)^2 \quad \text{with } \frac{1}{w}\frac{a(t_F)}{a(t_d)} \leq \tilde{f} \leq \frac{1}{w}$$

  where $\tilde{f} = (1 + z_d) f$.
- UV tail of GWs from collapse WBS structures

$$\Omega_{\rm GW}^{\rm col}(\tilde{f}, t_d) = \frac{\sigma w}{m_{\rm Pl}^2}(\tilde{f} w)^{-1}, \quad \text{with } \tilde{f} \geq \frac{1}{w} \, .$$

- The gravitational wave spectrum is given by

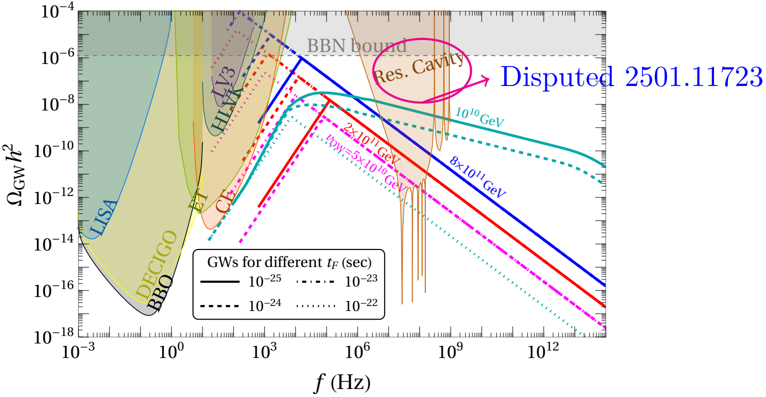
$$\Omega_{\rm GW}(f) = \mathcal{G}(z_d)\Omega_{r,0}\Omega_{\rm GW}(\tilde{f}, t_d),$$

  where $\Omega_{r,0} = 9.1476 \times 10^{-5}$ and $\mathcal{G}(z) = \frac{g_*(z)}{g_*(0)}\frac{g_{*s}(0)^{4/3}}{g_{*s}(z)^{4/3}}$.

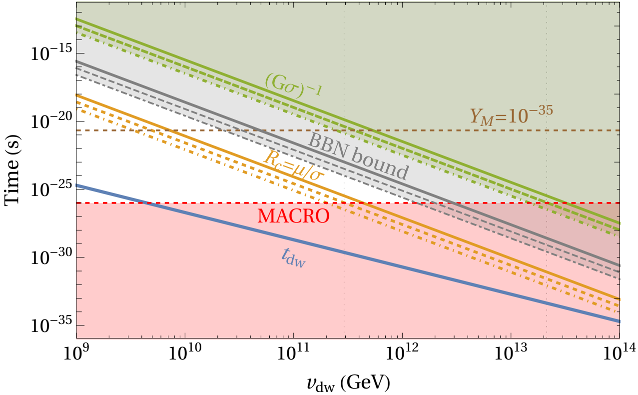
RM, Shafi, PhysRevD.111.075027

Gravitational wave signatures of C-parity

- Decay of the string-wall network emits gravitational waves.
- Expected frequency range: 10^2 - 10^5 Hz, depending on the horizon reentry time t_F for C-strings and GUT monopoles.



C-parity: GUT monopoles and GWs



RM, Shafi, PhysRevD.111.075027

4 *Summary*

Summary

- The breaking patterns involving C or D parity of $SO(10)$ can form wall bounded by strings along with the GUT monopoles.
- These walls bounded by strings can decay into gravitational wave background at a higher frequency ($f \sim 10^2 - 10^5$ Hz) with a partially inflated flux of monopoles ($Y_M \lesssim 10^{-27}$).
- The gravitational wave background can be observed in the proposed experiments along with an observable flux of GUT monopoles.

Thank You

Walls bounded by strings from the center of SO(10)

$$\underbrace{SO(10) \to \ldots \to \mathcal{G}_I}_{\text{rank}=5} \xrightarrow[M_R(\nu_L^C)]{\langle \Phi \in \overline{126} \rangle \gtrsim 10^{11}\ \text{GeV}} \underbrace{\text{SM} \times \mathbb{Z}_2 \xrightarrow[\simeq 10^2 - 10^5\ \text{GeV}]{\langle S \in 16 \rangle} \text{SM}}_{\text{rank}=4}$$

- $\langle 126_H \rangle \gtrsim 10^{11}$ GeV leaves an unbroken $\mathbb{Z}_2$ and therefore generates topologically stable cosmic strings.

  Kibble, Lazarides, Shafi (1982)

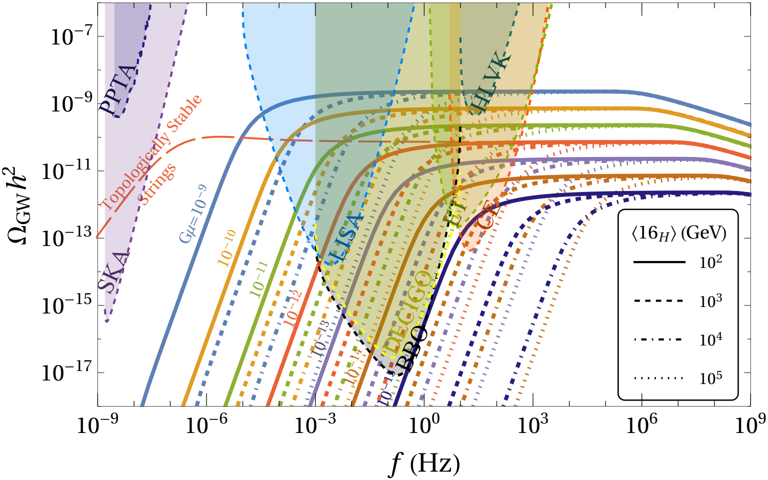
- The VEV $\langle 16_H \rangle \simeq [10^2, 10^5]$ GeV, breaks this $\mathbb{Z}_2$ symmetry, which leads to the formation of domain walls bounded by strings.

  RM, Park, Shafi, Phys.Lett.B **845** (2023) 138127

  Lazarides, Shafi, Tiwari JHEP 05 (2023) 119

- String has a tension $\mu \sim \langle 126_H \rangle^2$ and wall has a tension $\sigma \sim \langle 16_H \rangle^3$.

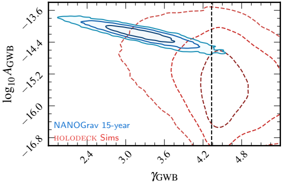
GWs from WBS and observational prospects



RM, Park, Shafi, Phys.Lett.B **845** (2023) 138127

Evidence of GWB in PTA

EPTA NANOGrav PPTA InPTA



Buchmuller, Domcke, Schmitz (2307.04691);

Antusch, Hinze, Saad, Steiner (2307.04595);

Lazarides, **RM**, Moursy, Shafi (2308.07094);

Fu et. al. (2308.05799); **RM**, Park (2308.11439)

RM, Moursy, Shafi (2409.13584);

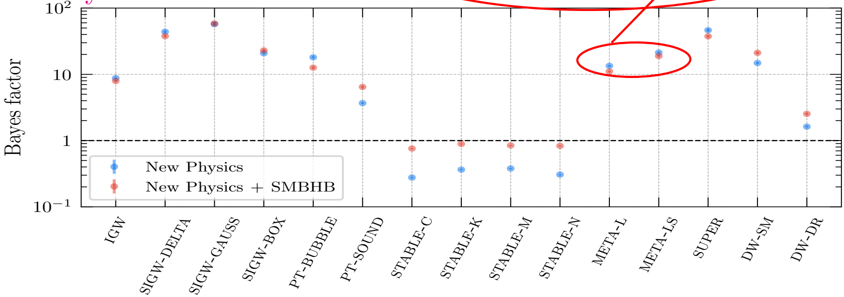
Antusch, Hinze, Saad (2503.05868); ...

Superheavy metastable strings with $G\mu \sim 10^{-6}$ and $\sqrt{\kappa} \sim 8$.

Mergers of SMBHBs?
2306.16213, 2306.16214, 2306.16215...

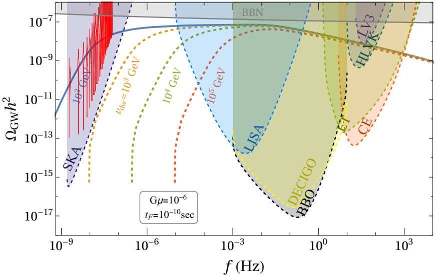
Cosmological Origin?

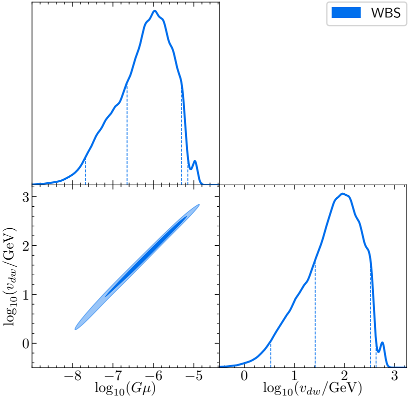
New Physics? NG15:2306.16219...



WBS and NANOGrav data

Parameters	Bayesian Credible Intervals	
	68%	95%
$\log_{10}(G\mu)$	$[-6.65, -5.31]$	$[-7.68, -5.15]$
$\log_{10}(v_{\rm dw}/\text{GeV})$	$[1.41, 2.52]$	$[0.53, 2.63]$





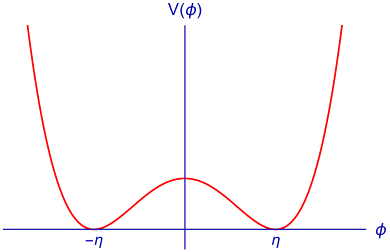
Lazarides, **RM**, Shafi, Phys. Rev. D **108** (2023) 095041

Back up slides

Prediction of topological defects

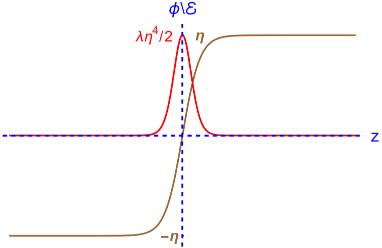
- Topological defects may appear during the SSB of a group $\mathcal{G}$ down to its subgroup $\mathcal{H}$.
- Non-trivial homotopy group $\Pi_k(\mathcal{M})$ of the vacuum manifold ($\mathcal{M} = \mathcal{G}/\mathcal{H}$) indicates formation of topological defects.
- Various types of topological defects which can be formed are : domain walls (k = 0), cosmic strings (k = 1), monopoles (k = 2) etc. $SU(N)_X \to U(1)_X$: Monopoles, $U(1) \to Z_N$: Strings
- GUTs predict topologically stable magnetic monopole that carries one unit $(2\pi/e)$ of Dirac magnetic charge.
- $SO(10)$ predicts formation of composite structures depending on the symmetry breaking patterns to the SM.

Example : domain wall

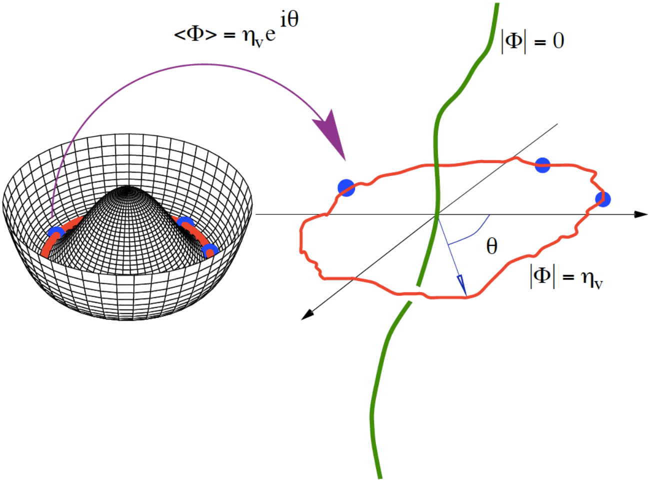


- $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{\lambda}{4}(\phi^2 - \eta^2)^2$
- Vacuum manifold consists of two disconnected elements : $\langle \phi \rangle = \pm\eta$: $\Pi_0(\mathcal{M}) = \mathbb{Z}_2$.
- Boundary conditions : $\phi \to \pm\eta$ as $z \to \pm\infty$.

- Stationary solution : $\phi(z) = \eta \tanh\left(\sqrt{\frac{\lambda}{2}}\eta z\right)$.
- Energy density : $\mathcal{E} = \frac{\lambda\eta^4}{2}\text{sech}^4\left(\sqrt{\frac{\lambda}{2}}\eta z\right)$.
- Energy per unit area : $\frac{2\sqrt{2}}{3}\sqrt{\lambda}\eta^3$ on xy plane $\Rightarrow$ $\boxed{\textit{Domain Wall}}$



Cosmic string



Vachaspati et. al. arXiv:1506.04039

Cosmic string network

- String tension $\mu \simeq \pi v^2$, v is the VEV that form the string.

- Strings inter-commute, form loops, radiate GWs and the evolution of the network enters a 'scaling' regime.

- Scaling energy density $\rho_s \sim \mu/t^2$. Critical density: $\rho_c \sim 1/Gt^2$ in RD and MD.

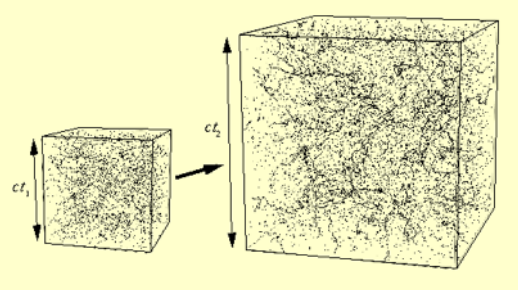


Image source: ctc.cam.ac.uk

Kibble, NPB 252 (1985) 227; Vachaspati, Vilenkin PRD 31 (1985) 3052; Bennett, Bouchet, PRL 60 (1988) 257 ...

Strings and gravitational waves

- Loops of initial length $\boxed{l_i = \alpha t_i}$ $(\alpha \simeq 0.1)$ decay via emission of gravity waves. Blanco-Pillado, Olum, Shlaer, Phys. Rev. D **89** (2014) 023512; Wachter, Blanco-Pillado, Olum arXiv:2411.16590

$$\frac{dE^{(k)}_{\rm GW}}{dt} = \Gamma_k G\mu^2; \quad \Gamma_k \propto k^{-n} \quad \text{with } n = \begin{cases} 4/3 & \text{cusps} \\ 5/3 & \text{kinks} \\ 2 & \text{kink-kink collisions.} \end{cases}$$

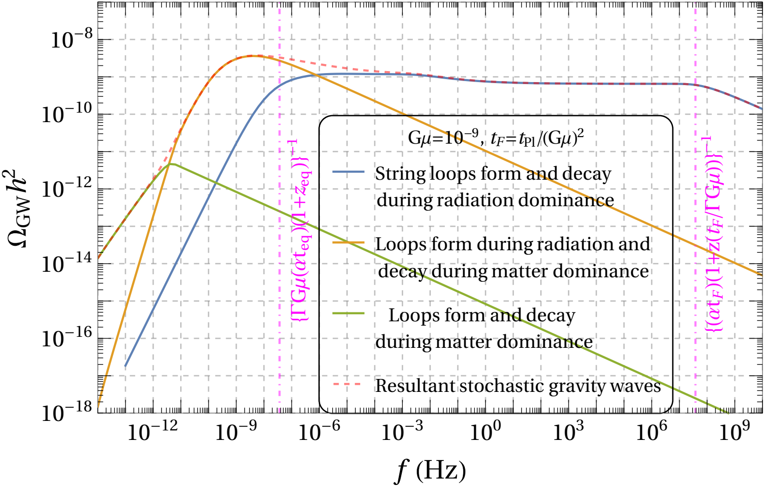
- The redshifted frequency of a normal mode k, emitted at time $\tilde{t}$, as observed today, is given by Vilenkin, Shellard, 1994, CUP

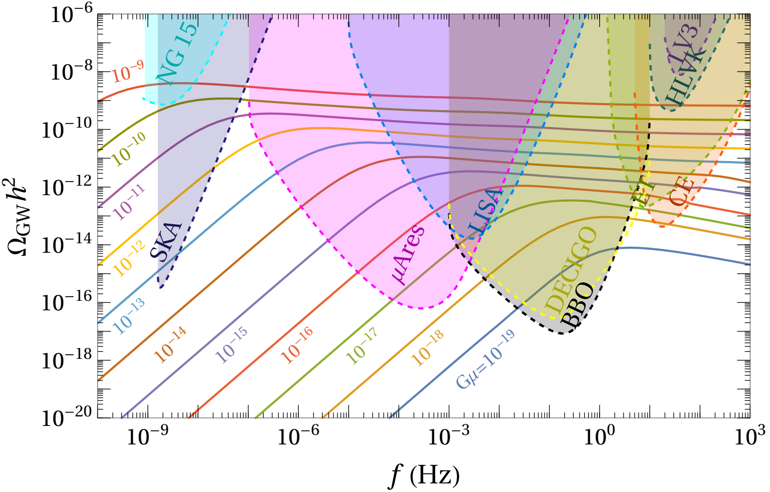
$$f = \underbrace{\frac{a(\tilde{t})}{a(t_0)}}_{\text{Redshift}} \frac{2k}{\alpha t_i - \Gamma G\mu(\tilde{t} - t_i)}, \quad \text{with} \quad \Gamma = \sum \Gamma_k \sim 50$$

Redshift

$\frac{dl}{dt} = -\Gamma G\mu \Rightarrow$ Loop size at time $\tilde{t}$

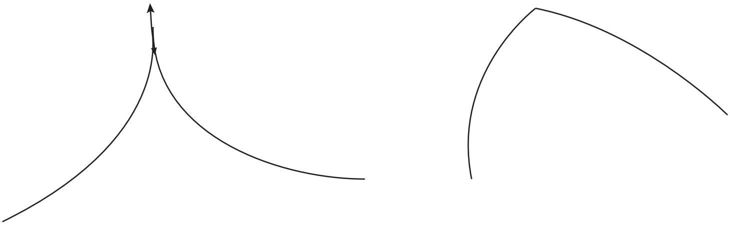
Stochastic gravitational wave background





$\Omega_{\rm GW} h^2$
f (Hz)
NG 15
SKA
μAres
LISA
DECIGO
BBO
ET
CE
HLVK
LV3
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10^{-16}
10^{-17}
10^{-18}
$G\mu=10^{-19}$

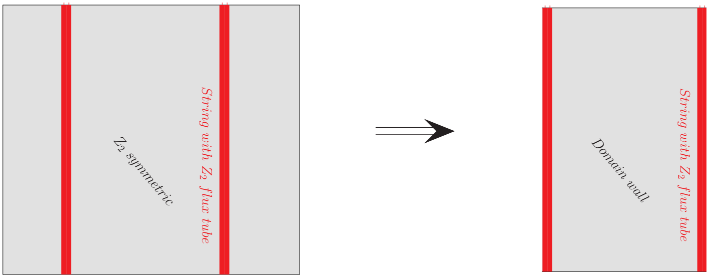
Cusp and Kink



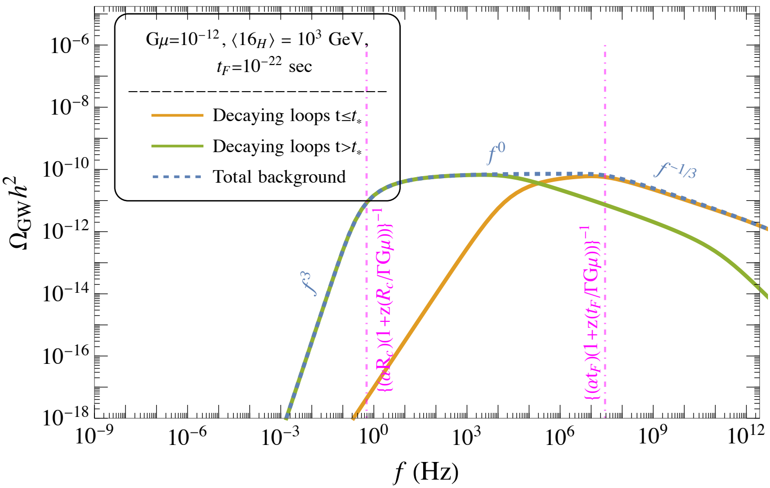
Formation of domain walls bounded by strings

- Consider $G \xrightarrow{M_I} H \otimes Z_2 \xrightarrow{M_{II}} H$
  with G being simply connected and $\Pi_1(G/H) \cong \Pi_0(H) = I$.

- Domain walls formed at M_{II} connect strings formed at M_I.



Gravitational waves from WBS



RM, Park, Shafi, Phys.Lett.B **845** (2023) 138127

Pulsar Timing Arrays

- Pulsars are rapidly spinning neutron stars with a strong magnetic field $\Rightarrow$ Radiate beam of radio waves.
- Repeating pulses are observed as the radio beam intersects the observers periodically.
- Millisecond pulsar (MSP) produces exceedingly stable and regular pulse profile $\Rightarrow$ "Perfect Clock".

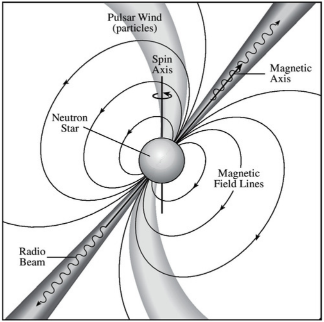
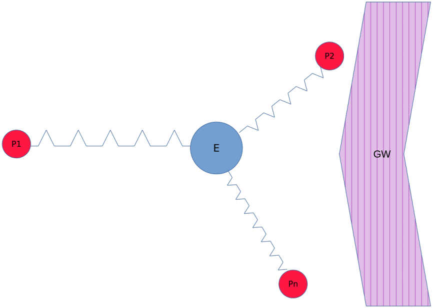


Image source: K.R. Lang, NASA's Cosmos

Pulsar Timing Arrays



- Measurement of the time of arrival (ToA) of pulses can reveal tiny distortion of spacetime fabric due to gravity waves (GWs) $\Rightarrow$ Pulsar timing!

Image source: Wikipedia

- Difference between observed ToA and the expected ToA from timing model gives time residual.

- Time residual contains information about other signals like GWs.

Pulsar Timing Arrays

- Impossible to distinguish between GWs signal and other source of signal in the timing residual of a single pulsar.
- Need correlations between the timing residuals of different pulsars $\Rightarrow$ Pulsar Timing Array (PTA).
- Gravity waves generate unique quadrupolar correlations between timing residuals of pulsar pairs.
- Correlations depend on the angular separations between the pulsar pairs and follow the Hellings and Downs correlation curve. **APJ. 265, L39 (1983)**

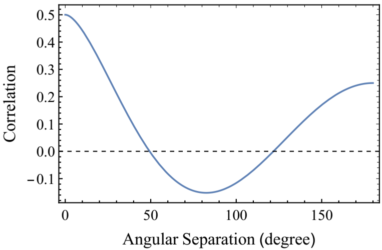


Figure: Hellings and Downs curve.