Constraining dark matter from strong phase transitions in a Lmu-Ltau model: Implications for neutrino masses and muon *g*−2

Sarif Khan

Chung-Ang University, Seoul

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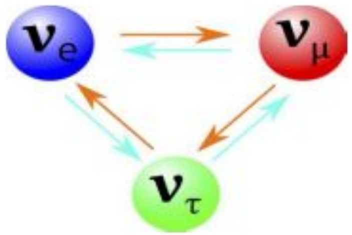
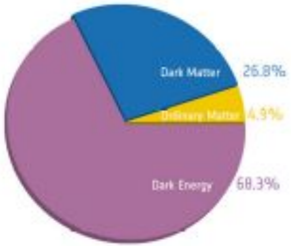
Problems in the SM

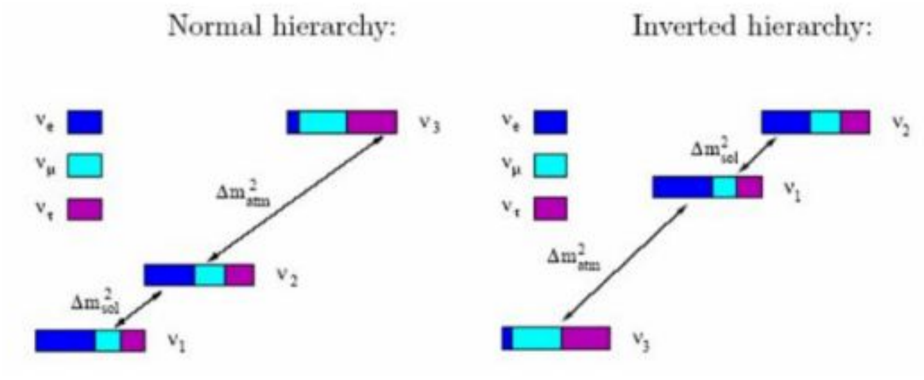
- SM fails to explain neutrino mass and mixings.
- SM doesn't have a DM candidate.
- SM can not explain the observed baryon asymmetry.
- The origin of smallness of the θ-parameter.





Neutrino Mass and open problems

- **Mass ordering unknown**
- **Which octant for θ_{23}**
- **What is the value of the CP-phase δ?**
- **Origin of small neutrino mass?**



Dirac or Majorana??

Neutrino Masses

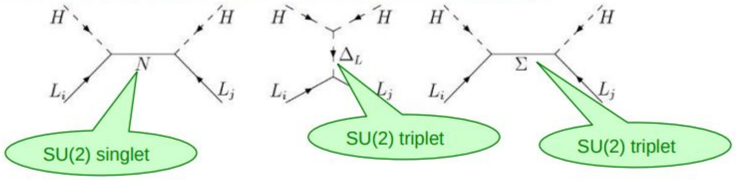
⇒Leptogenesis?

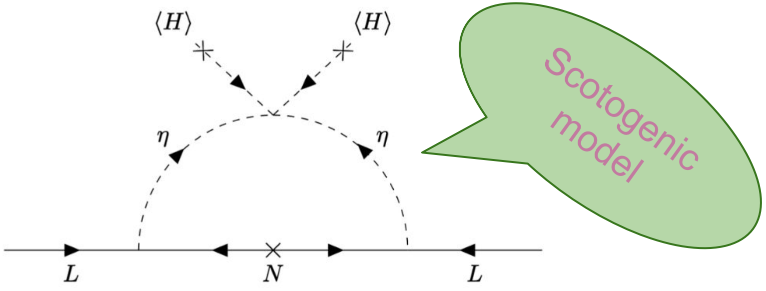
Majorana mass (Violate Lepton number)

e.g., Lepton No. violating interactions in BSM, R-parity violation in SUSY, etc.

$$\mathcal{L}_{Weinberg} = \frac{1}{2}\frac{1}{M_\nu} L_i L_j H H$$

See-saw (Add new fermions/scalars)





Particle Set up

Gauge Group	Baryonic Fields	Vector-like	Lepton Fields			Scalar Fields		
	(Q_L^i, u_R^i, d_R^i)	$\psi_L,\, \psi_R$	(L_L^e, e_R, N_R^e)	$(L_L^\mu, \mu_R, N_R^\mu)$	$(L_L^\tau, \tau_R, N_R^\tau)$	Φ	Φ'	Φ_{DM}
$U(1)_{L_\mu - L_\tau}$	0	q_ψ	0	1	-1	0	1	q_{DM}

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm scalar} + \mathcal{L}_\psi + \mathcal{L}_{Z'} + \mathcal{L}_N \,,$$

$$D_\mu X \equiv \left(\partial_\mu - i g_{Z'} Q(X) Z'_\mu\right) X,$$

$$\mathcal{L}_N = \sum_{i=e,\,\mu,\,\tau} \frac{i}{2} \bar{N}_i \gamma^\mu D_\mu N_i - \frac{1}{2} M_{ee}\, \bar{N}_e^c N_e - \frac{1}{2} M_{\mu\tau} \left(\bar{N}_\mu^c N_\tau + \bar{N}_\tau^c N_\mu\right)$$
$$-\frac{1}{2} h_{e\mu} (\bar{N}_e^c N_\mu + \bar{N}_\mu^c N_e) \Phi'^\dagger - \frac{1}{2} h_{e\tau} (\bar{N}_e^c N_\tau + \bar{N}_\tau^c N_e) \Phi'$$
$$- \sum_{i=e,\,\mu,\,\tau} y_i \bar{L}_i \tilde{\Phi} N_i + h.c.$$

$$\mathcal{M}_R = \begin{pmatrix} M_{ee} & \frac{v'}{\sqrt{2}} h_{e\mu} & \frac{v'}{\sqrt{2}} h_{e\tau} \\ \frac{v'}{\sqrt{2}} h_{e\mu} & 0 & M_{\mu\tau}\, e^{i\xi} \\ \frac{v'}{\sqrt{2}} h_{e\tau} & M_{\mu\tau}\, e^{i\xi} & 0 \end{pmatrix}, \qquad M_D = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}.$$

$$m_\nu \simeq -M_D\, M_R^{-1} M_D^T \,,$$
$$m_N \simeq M_R \,,$$

Mass matrix and PMNS

$$m_\nu = \frac{1}{2p}\begin{pmatrix} 2 f_e^2 M_{\mu\tau}^2 e^{i\xi} & -\sqrt{2} f_e f_\mu h_{e\tau} v_{\mu\tau} & -\sqrt{2} f_e f_\tau h_{e\mu} v_{\mu\tau} \\ -\sqrt{2} f_e f_\mu h_{e\tau} v_{\mu\tau} & \frac{f_\mu^2 h_{e\tau}^2 v_{\mu\tau}^2 e^{-i\xi}}{M_{\mu\tau}} & \frac{f_\mu f_\tau}{M_{\mu\tau}}(M_{ee} M_{\mu\tau} - p e^{-i\xi}) \\ -\sqrt{2} f_e f_\tau h_{e\mu} v_{\mu\tau} & \frac{f_\mu f_\tau}{M_{\mu\tau}}(M_{ee} M_{\mu\tau} - p e^{-i\xi}) & \frac{f_\tau^2 h_{e\mu}^2 v_{\mu\tau}^2 e^{-i\xi}}{M_{\mu\tau}} \end{pmatrix},$$

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}.$$

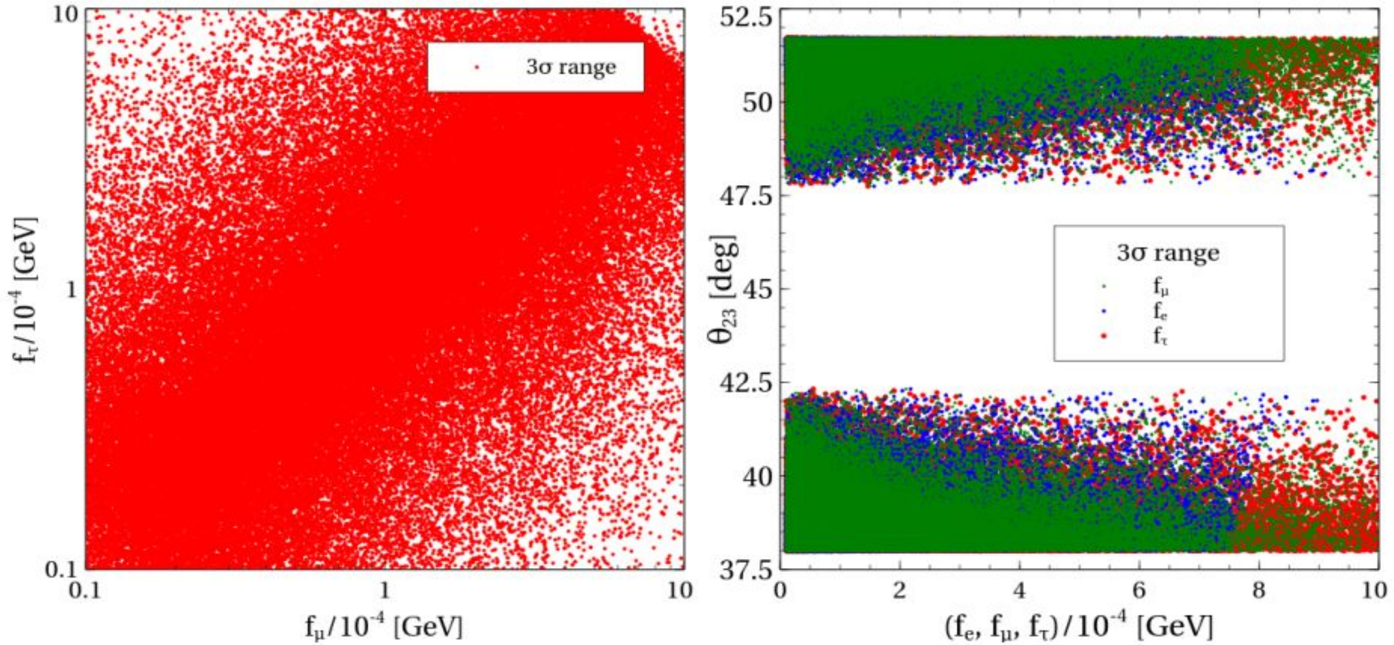
$$\begin{bmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{CP}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{CP}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{CP}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{CP}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{CP}} & c_{23} c_{13} \end{bmatrix},$$

$$\begin{array}{rcccl} 0 & \leq & \xi\,[\mathrm{rad}] & \leq & 2\pi\,, \\ 1 & \leq & M_{ee},\ M_{\mu\tau}\,[\mathrm{GeV}] & \leq & 10^4\,, \\ 1 & \leq & V_{e\mu},\ \ V_{e\tau}\ \,[\mathrm{GeV}] & \leq & 280\,, \\ 0.1 & \leq & \frac{(f_e,\ f_\mu,\ f_\tau)}{10^{-4}}\,[\mathrm{GeV}] & \leq & 10\,. \end{array}$$

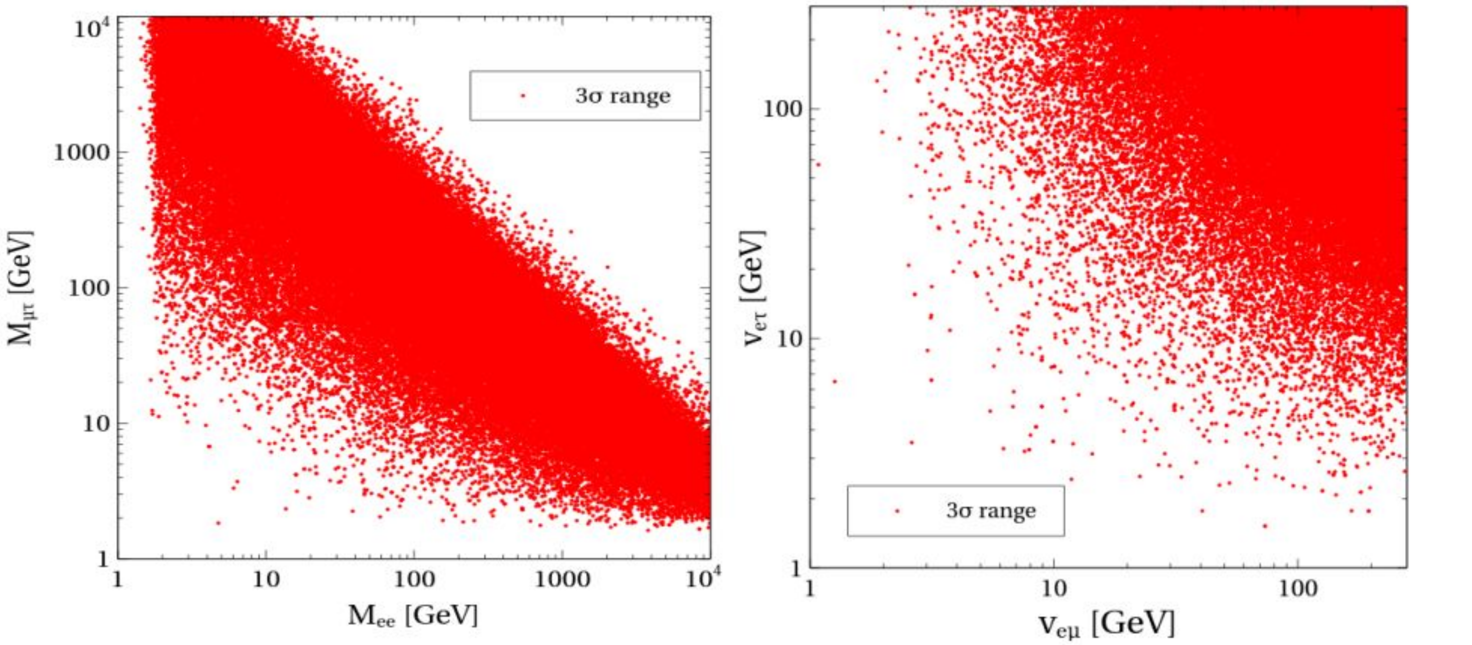
Global Fit

IC24 with SK atmospheric data	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 6.1$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.308^{+0.012}_{-0.011}$	$0.275 \rightarrow 0.345$	$0.308^{+0.012}_{-0.011}$	$0.275 \rightarrow 0.345$
$\theta_{12}/^\circ$	$33.68^{+0.73}_{-0.70}$	$31.63 \rightarrow 35.95$	$33.68^{+0.73}_{-0.70}$	$31.63 \rightarrow 35.95$
$\sin^2\theta_{23}$	$0.470^{+0.017}_{-0.013}$	$0.435 \rightarrow 0.585$	$0.550^{+0.012}_{-0.015}$	$0.440 \rightarrow 0.584$
$\theta_{23}/^\circ$	$43.3^{+1.0}_{-0.8}$	$41.3 \rightarrow 49.9$	$47.9^{+0.7}_{-0.9}$	$41.5 \rightarrow 49.8$
$\sin^2\theta_{13}$	$0.02215^{+0.00056}_{-0.00058}$	$0.02030 \rightarrow 0.02388$	$0.02231^{+0.00056}_{-0.00056}$	$0.02060 \rightarrow 0.02409$
$\theta_{13}/^\circ$	$8.56^{+0.11}_{-0.11}$	$8.19 \rightarrow 8.89$	$8.59^{+0.11}_{-0.11}$	$8.25 \rightarrow 8.93$
$\delta_{CP}/^\circ$	212^{+26}_{-41}	$124 \rightarrow 364$	274^{+22}_{-25}	$201 \rightarrow 335$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.49^{+0.19}_{-0.19}$	$6.92 \rightarrow 8.05$	$7.49^{+0.19}_{-0.19}$	$6.92 \rightarrow 8.05$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.513^{+0.021}_{-0.019}$	$+2.451 \rightarrow +2.578$	$-2.484^{+0.020}_{-0.020}$	$-2.547 \rightarrow -2.421$

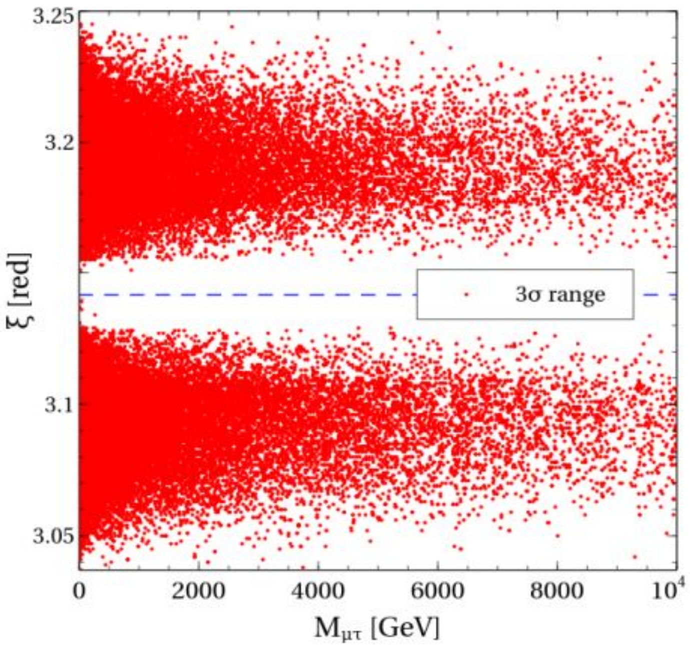
Allowed Region

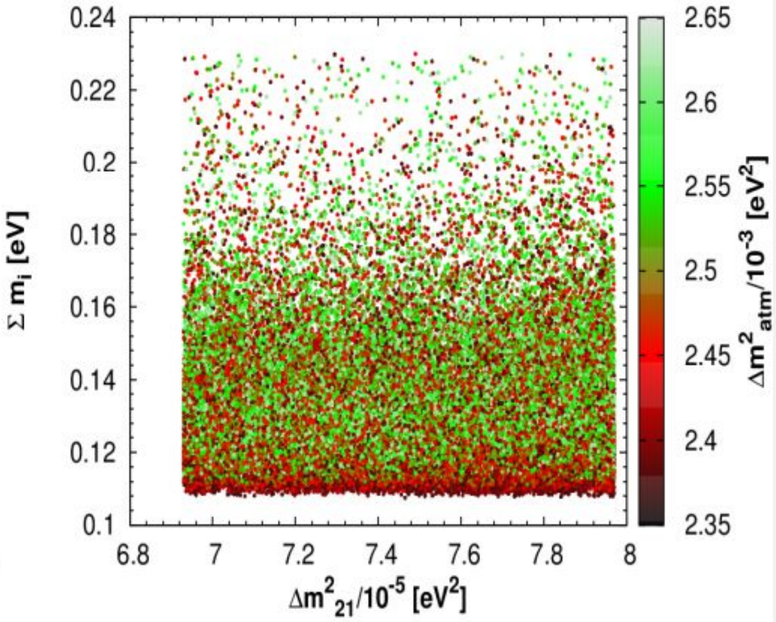


Allowed Region

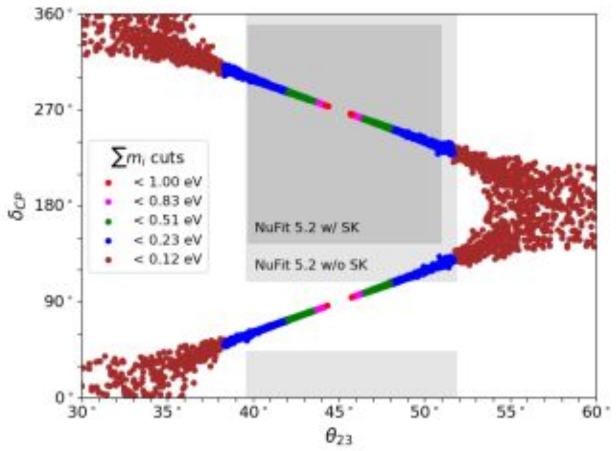


Allowed Region





Neutrino mass



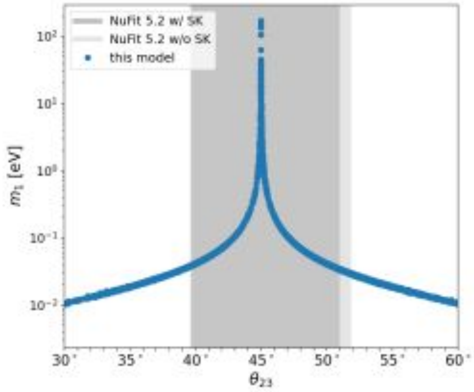
$$m_\nu = \frac{1}{2p}\begin{pmatrix} 2\, f_e^2 M_{\mu\tau}^2 e^{i\xi} & -\sqrt{2}\, f_e f_\mu\, h_{e\tau} v_{\mu\tau} & -\sqrt{2}\, f_e f_\tau\, h_{e\mu} v_{\mu\tau} \\ -\sqrt{2}\, f_e f_\mu\, h_{e\tau} v_{\mu\tau} & \dfrac{f_\mu^2\, h_{e\tau}^2\, v_{\mu\tau}^2\, e^{-i\xi}}{M_{\mu\tau}} & \dfrac{f_\mu\, f_\tau}{M_{\mu\tau}}(M_{ee}\, M_{\mu\tau} - p\, e^{-i\xi}) \\ -\sqrt{2}\, f_e f_\tau\, h_{e\mu} v_{\mu\tau} & \dfrac{f_\mu\, f_\tau}{M_{\mu\tau}}(M_{ee}\, M_{\mu\tau} - p\, e^{-i\xi}) & \dfrac{f_\tau^2\, h_{e\mu}^2\, v_{\mu\tau}^2\, e^{-i\xi}}{M_{\mu\tau}} \end{pmatrix}$$

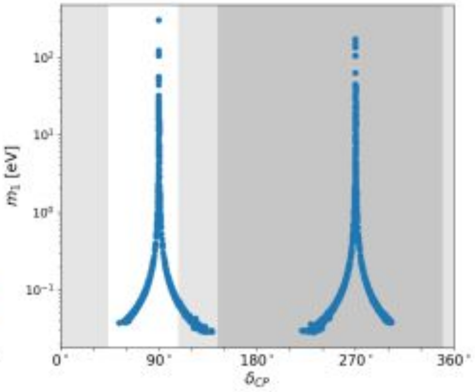
$$p = h_{e\mu} h_{e\tau} v'^2 - M_{ee} M_{\mu\tau} e^{i\xi}$$

$$\delta_{CP}$$

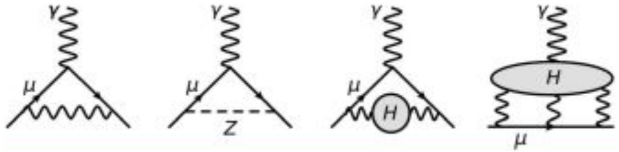
$$\sin\delta_{CP} = \frac{\mathrm{Im}\left[U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}\right]}{J_{CP}^{\max}}.$$

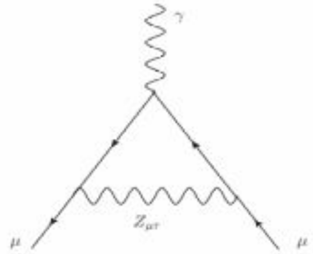
$$\cos\delta_{CP} = \frac{\mathrm{Re}\left[U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}\right] + s_{12}^2 s_{13}^2 c_{13}^2 s_{13}^2}{J_{CP}^{\max}}.$$





Muon g-2





$$\Delta a_\mu(Z_{\mu\tau}) = \frac{g_{\mu\tau}^2}{8\pi^2}\int_0^1 dx \frac{2x(1-x)^2}{(1-x)^2 + rx},$$

- The Magnetic moment of a particle with spin is given by $\mu = g\frac{e}{2m}\mathbf{S}$
- SM contribution to muon (g-2) includes QED, HVP, HLbL and EW Processes
- Among the four contribution HVP comes from the experimental measurement.
- SM predicts with all the contribution $a_\mu^{SM} = 116591810(43) \times 10^{-11}(0.37ppm)$
- Expt. Measurements at CERN, BNL and FNAL delivers world average value $a_\mu^{Exp} = 116592061(41) \times 10^{-11}(0.35ppm)$
- Difference between the experimental and theoretical value predicts

$$\Delta a_\mu = a_\mu^{Exp} - a_\mu^{SM} = (251 \pm 59) \times 10^{-11}$$

Discrepancy in VHP

- Recent measurement in the Lattice computation measure the VHP contribution which differ from the experimental value by 2.1 sigma.
- This measuement reduces the significance of the discrepancy between the experimental and theoretical value to 1.5 sigma.
- New measurement of VHP contribution by CMD-3 experiment reduces the theoretical and experimental gap to 2.4 sigma.
- The new measurement differs from the previous measurement of the same experiment as well as the other experiments.
- We need further confirmation between any conclusion on the muon g-2 anomaly
- The present work tried to explain the muon g-2 discrepancy observed at the FNAL and the theoretical value

Gravitational Waves

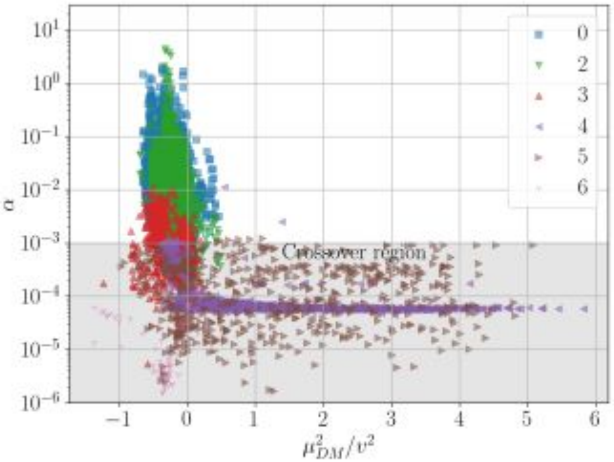
$$V_T(\phi,\phi',\chi,T) = \frac{T^4}{2\pi^2}\sum_i d_i J_{\mp}\left(\frac{m_i(\phi,\phi',\chi)}{T}\right)$$

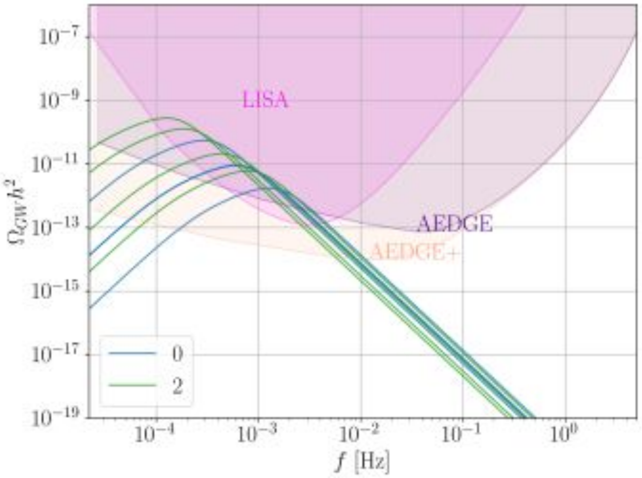
$$V_{\rm CW}(\phi,\phi',\chi) = \sum_i (-1)^{F_i}\frac{d_i}{64\pi^2}\left[m_i^4(\phi,\phi',\chi)\left(\log\frac{m_i^2(\phi,\phi',\chi)}{m_{0i}^2}-\frac{3}{2}\right)+2m_i^2(\phi,\phi',\chi)m_{0i}^2\right],$$

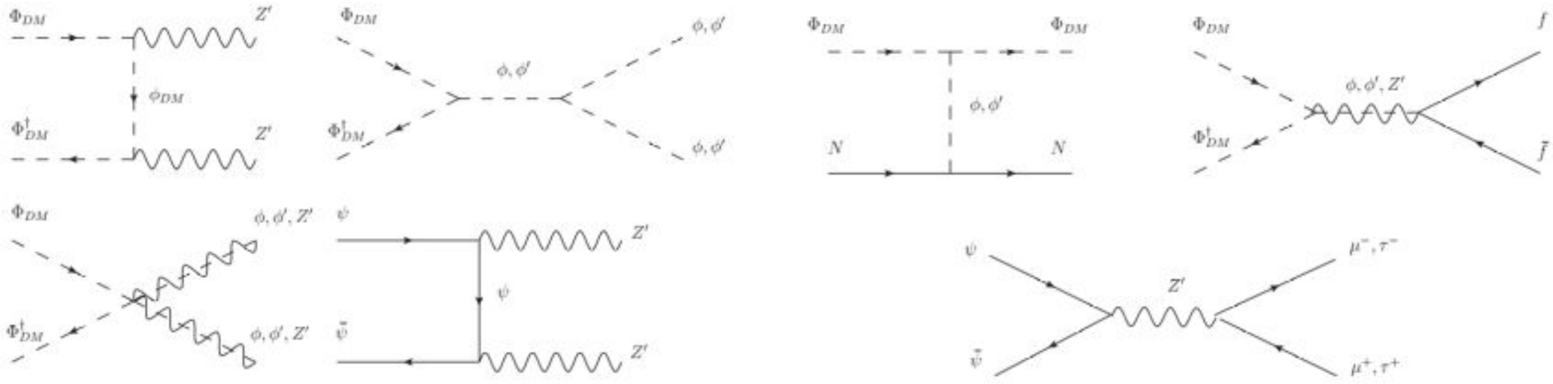
$$V_{eff}(\phi,\phi',\chi,T) = V_0(\phi,\phi',\chi) + V_{CW}(\phi,\phi',\chi) + V_T(\phi,\phi',\chi,T).$$

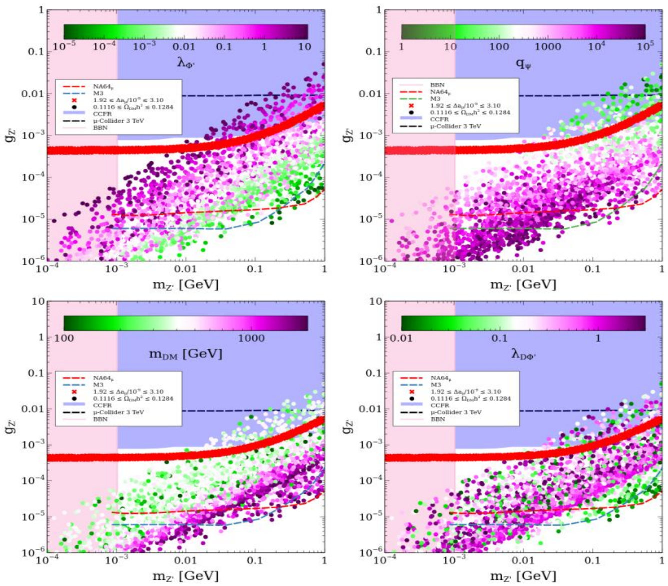
$$\begin{aligned}V_0(\Phi,\Phi',\Phi_{\rm DM}) = & V_{\rm SM}(\Phi) + \mu^2_{DM}\Phi^\dagger_{DM}\Phi_{DM} + \lambda_{DM}(\Phi^\dagger_{DM}\Phi_{DM})^2 \\ & + \mu^2_{\phi'}\Phi'^\dagger\Phi' + \lambda_{\phi'}(\Phi'^\dagger\Phi')^2 \\ & + \lambda_{D\phi}(\Phi^\dagger_{DM}\Phi_{DM})(\Phi^\dagger\Phi) + \lambda_{D\phi'}(\Phi^\dagger_{DM}\Phi_{DM})(\Phi'^\dagger\Phi') \\ & + \lambda_{\phi\phi'}(\Phi'^\dagger\Phi')(\Phi^\dagger\Phi),\end{aligned}$$

direction	All	$\phi-\phi'$	$\phi-\chi$	$\phi'-\chi$	ϕ	ϕ'	χ
label	0	1	2	3	4	5	6

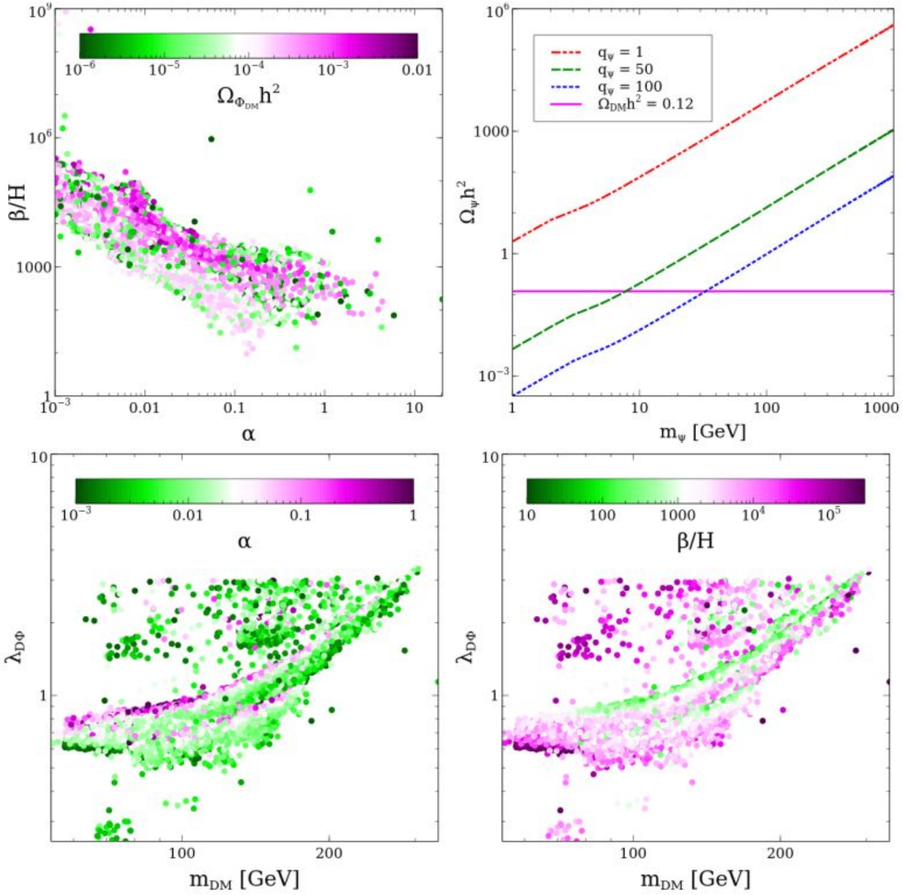




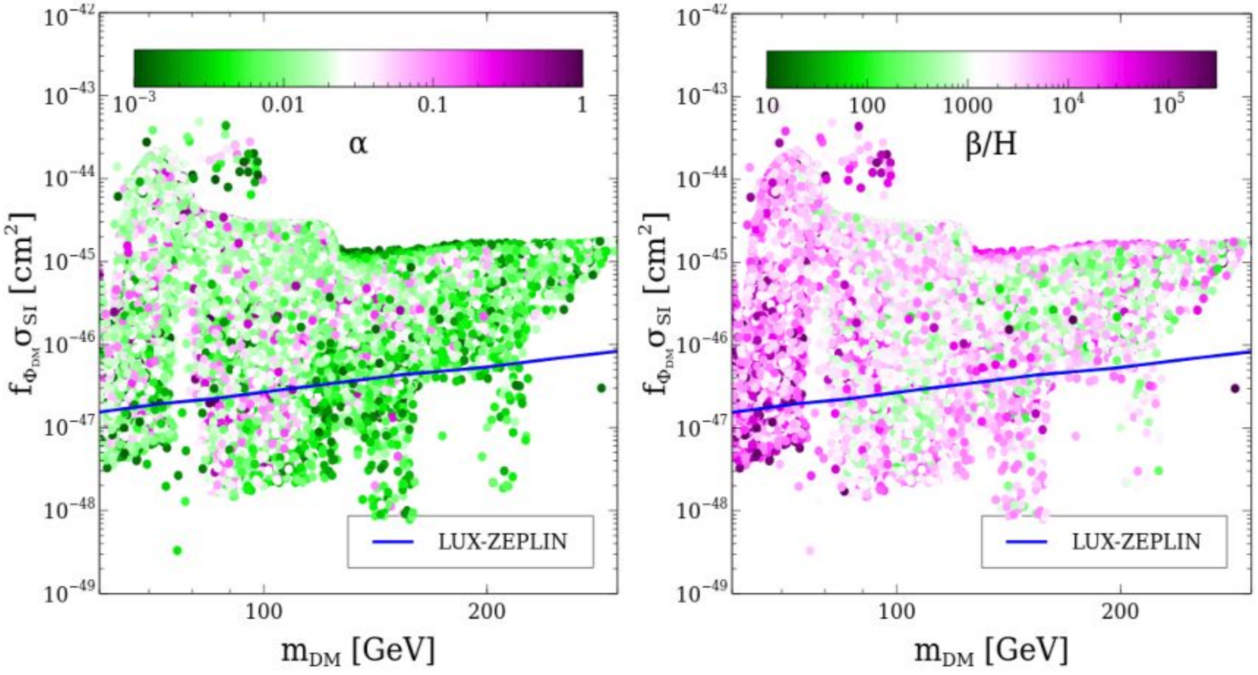




- Relic density bound
- Direct detection bound
- Indirct detection bound
- Collider bounds using HB and HS

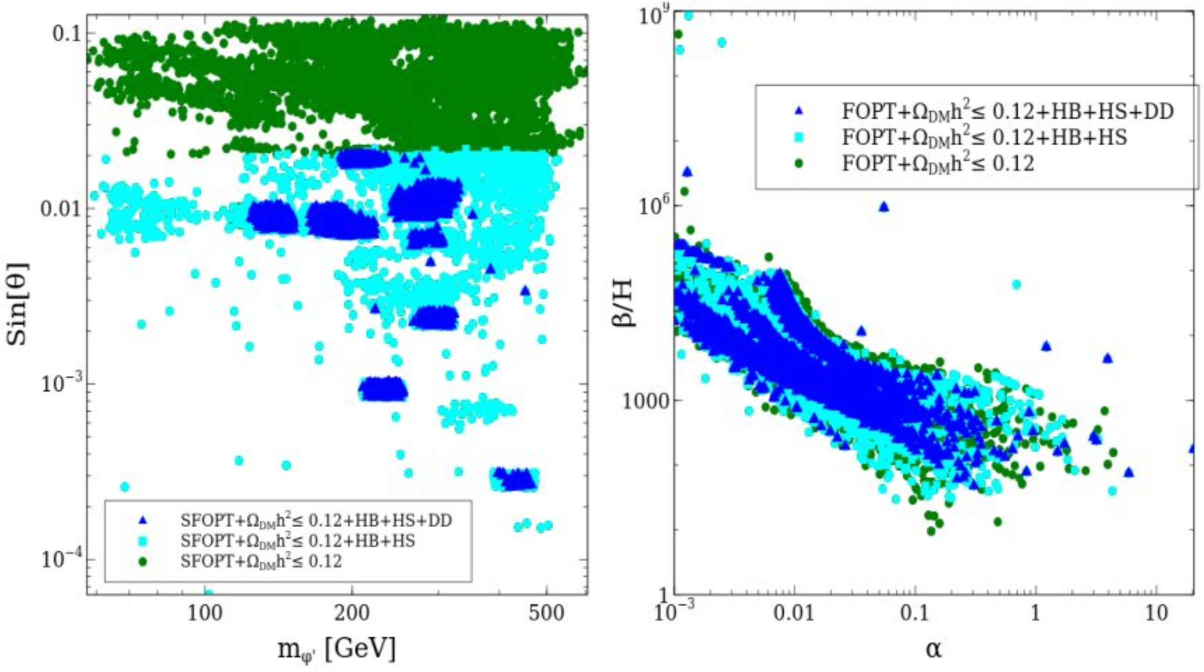


- ULP shows the fraction of scalar DM which exhibits we can not go beyond DM RD 0.01
- URP shows that fermion DM can accommodate the rest of the component
- In the LLP, we have shown in the colorbar the GW parameters and for SFOPT we need quartic coupling larger



- In both the plots, we can see a large portion of parameter space is in conflict with the DMDD.
- We have chcked if DM RD is 1e-6 to 1e-5, only then it will be allowed from the DD which demand a second component DM.

$$\frac{\Gamma_{\phi\to Z'Z'}}{\Gamma_\phi} \leq 10.7\% \Rightarrow \sin\theta < 0.02,$$

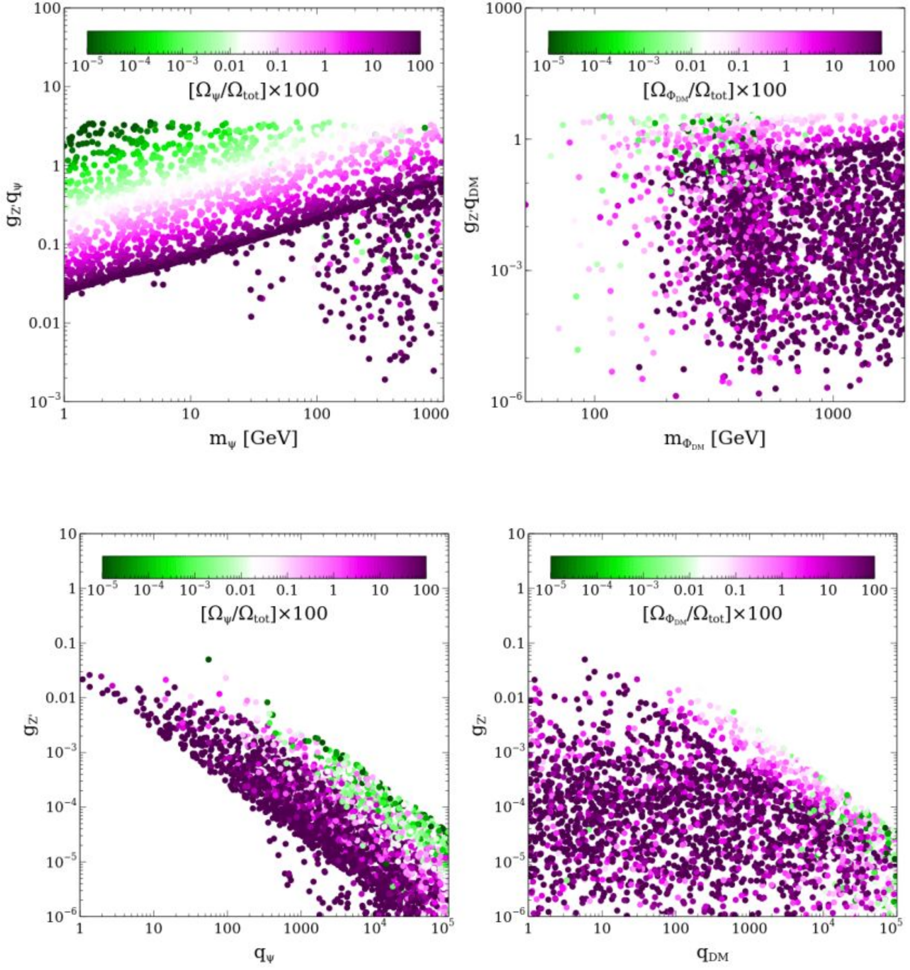


- In the LP we see sharp upper cut on the mixing angle which is 0.02 coming from the Higgs invisible decay
- In the RP, we can see even after all the bounds we have more or less all the range of alpha and beta are allowed.

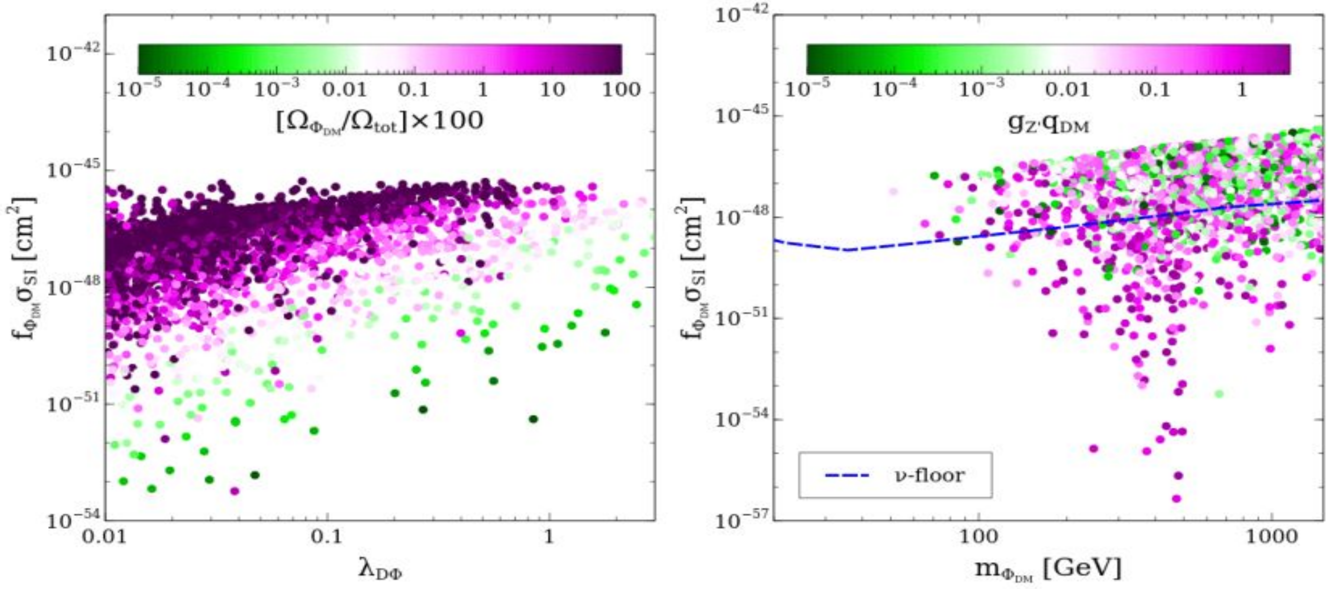
Conclusion

- The present model can explain the total amount of DM, neutrino mass, muon g-2 and SFOPT.
- DD demands the 1e-6 to 1e-5 dark matter relic density contribution but the rest of the component can come from the fermion DM.
- We find that the atmospheric angle 45 degree is not possible due to resonance nature in the lightest neutrino mass.
- We find some of the regimes which can be explored in future by GW are already ruled out by DD bound.
- Our model prefers Dirac CP phase peaked around 270 degree and 90 degree which is outside the current bound.

Thank You



- In the ULP, we see DM RD is inversely proportional qz q_psi but for mass above 100 GeV we see psi annihilation to sclar DM starts contributing
- In the URP, we do not see such correlation because Higgs mediated processes also contributes significantly
- In the LLP, we see night correlation between q_psi and g_Z but for the LRP we do not see such behaviour.



- LP shows the correlation between the quartic coupling and the SIDD times fraction of DM.
- If we move towards x-axis then we see reduction in DM fraction from the color variation due to the increment in the quartic coupling.
- In the RP, we have changed the x-axis with DM mass and random values of g_Z q_DM as see from the color variation.