PROBING THE 3+1 NEUTRINO MODEL IN THE SHiP EXPERIMENT

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ABSTRACT

We present the three-flavor sensitivity study of the SHiP experiment to sterile neutrino oscillations. [1] To find a sensitivity, a Feldman-Cousins method with a parametric bootstrap for systematics are used. Compared to the current near detector at 27 m (NSND), adding a far detector at 120 m (FSND) boosts sensitivity to $|U_{\alpha 4}|^2$ by up to an order of magnitude. After five years at the CERN SPS (2×10^{20} PoT), SHiP can reach $|U_{\alpha 4}|^2 \sim$ few $\times 10^{-3}$ around $\Delta m^2_{41} \sim 10^3\,\mathrm{eV}^2$ if we include FSND.

Motivation

- Anomalies (LSND, MiniBooNE, reactor, gallium) could hint at an additional sterile neutrino state.
- **SHiP** provides an intense, high-energy mixed-flavor beam with negligible muon & hadron background.
  $\Rightarrow$ short baseline experiment
- With robust statistical methods, we find comprehensive three-flavor sensitivity of the SHiP experiment to sterile neutrino oscillations.
- Dual-baseline option (NSND at 27 m + FSND at 120 m) is also investigated.

3+1 Model Essentials

$$|\nu_\alpha\rangle = \sum_{i=1}^{4} U^*_{\alpha i} |\nu_i\rangle\,, \quad (\alpha = e, \mu, \tau, s) \tag{1}$$

$$P_{\alpha\beta} \simeq \begin{cases} 1 - 4|U_{\alpha 4}|^2(1 - |U_{\alpha 4}|^2)\sin^2\left(\dfrac{\Delta m^2_{41} L}{4E_\nu}\right) & \alpha = \beta \\ 4|U_{\alpha 4}|^2|U_{\beta 4}|^2 \sin^2\left(\dfrac{\Delta m^2_{41} L}{4E_\nu}\right) & \alpha \neq \beta \end{cases} \tag{2}$$

SND location at SHiP

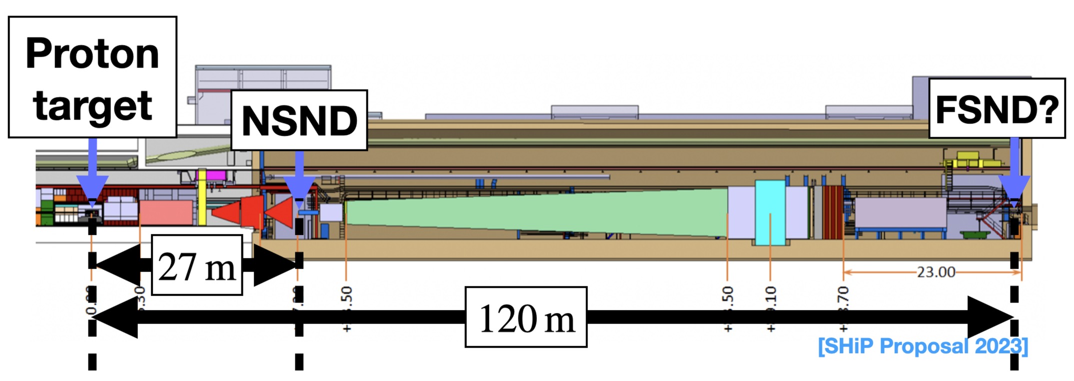

Figure 1. Simple layout of SHiP, with indication of SND locations and distances from the proton target.

data estimation

CC DIS signal $s_{\alpha,i}$ is

$$s_{\alpha,i} = \sum_{\beta}^{e,\mu,\tau}(1 + \phi_\beta + \phi_{\beta,i}) \int_{E_{\mathrm{start},i}}^{E_{\mathrm{end},i}} dE_\nu \int_{L_0}^{L_0+L_{\mathrm{d}}} dL \int dE_{\mathrm{rec}} \times \varepsilon_\alpha P_{\beta\alpha} \frac{d^2 N_\beta}{dE_\nu dL} \frac{\sigma_{\nu_\alpha A}}{\sigma_{\nu_\beta A}} \rho(E_{\mathrm{rec}}|E_\nu) \tag{3}$$

- ϕ_β, $\phi_{\beta,i}$: overall and shape flux nuisance terms
- N_β: # of β neutrinos crossing SND (3-flavor model)
- ε_α, $\sigma_{\nu_\alpha A}$: detection efficiency and cross-section
- $(E_{\mathrm{start},i}, E_{\mathrm{end},i})$: energy range of i-th bin
- L_0: baseline length
- L_{d}: detector length
- E_{rec}: reconstructed neutrino energy
- $\rho(E_{\mathrm{rec}}|E_\nu)$: energy response (Gaussian, $\sigma = 0.2E_\nu$) [2]

Both detection efficiencies & background ratio are referred from OPERA [3] (e, μ) and Ref. [4] with $\phi_{\tau-h} \geq 2.8\,\mathrm{rad}$ (τ)

$$\varepsilon_e = 30\%, \quad \varepsilon_\mu = 40\%, \quad \varepsilon_\tau = 10\%, \tag{4}$$

and the background $b_{\alpha,i}$ is

$$b_{e,i} = b_{\mu,i} = 0$$
$$b_{\tau,i} = (1 + \phi_\mu + \phi_{\mu,i}) \int_{E_{\mathrm{start},i}}^{E_{\mathrm{end},i}} dE_\nu \int_{L_0}^{L_0+L_{\mathrm{d}}} dL \int_0^\infty dE_{\mathrm{rec}} \times R^{-1}_{\mathrm{s/b}} \varepsilon_\tau P_{\mu\mu} \frac{d^2 N_\tau}{dE_\nu dL} \rho(E_{\mathrm{rec}}|E_\nu). \tag{5}$$

where the background of electron and muon channels are ignored, due to 95% (e) and 99% (μ) purities. [3]

Statistical Framework

Confidence region from given data $\boldsymbol{D}$ & point est. of $\boldsymbol{\phi}$ from auxiliary measurements:

$$\mathrm{CR}(\boldsymbol{D}, \boldsymbol{\varphi}; \alpha) = \{\boldsymbol{\theta} | \mathfrak{p}(\boldsymbol{\theta}; \boldsymbol{D}, \boldsymbol{\varphi}) > \alpha\} \tag{6}$$

The p-value $\mathfrak{p}$ is defined as a CDF of a test statistic λ:

$$1 - \mathfrak{p}(\boldsymbol{\theta}; \boldsymbol{D}, \boldsymbol{\varphi}) = \int_0^{\lambda(\boldsymbol{D},\boldsymbol{\varphi},\boldsymbol{\theta})} \rho(\lambda'|\boldsymbol{\theta}, \boldsymbol{\phi}) d\lambda'. \tag{7}$$

PDF of a test statistic:

$$\rho(\lambda'|\boldsymbol{\theta}, \boldsymbol{\phi}) = \int \delta[\lambda' - \lambda(\boldsymbol{O}, \boldsymbol{\varphi}', \boldsymbol{\theta})] \rho(\boldsymbol{O}, \boldsymbol{\varphi}'|\boldsymbol{\theta}, \boldsymbol{\phi}) d\boldsymbol{O}\, d\boldsymbol{\varphi}' \tag{8}$$

$\boldsymbol{O}$ & $\boldsymbol{\varphi}'$: Sample of $\boldsymbol{D}$ & $\boldsymbol{\varphi}$
The sample distribution $\rho(\boldsymbol{O}, \boldsymbol{\varphi}'|\boldsymbol{\theta}, \boldsymbol{\phi})$:

$$\rho(\boldsymbol{O}, \boldsymbol{\varphi}'|\boldsymbol{\theta}, \boldsymbol{\phi}) = \prod_{\alpha,i} \mathrm{Pois}(O_{\alpha,i}|s_{\alpha,i} + b_{\alpha,i}) \times \rho(\boldsymbol{\varphi}'|\boldsymbol{\phi}) \tag{9}$$

The profile likelihood ratio [5, 6] is chosen as λ:

$$\lambda(\boldsymbol{O}, \boldsymbol{\varphi}, \boldsymbol{\theta}) \equiv -2 \log \frac{\max_{\boldsymbol{\phi}} \rho(\boldsymbol{O}, \boldsymbol{\varphi}|\boldsymbol{\theta}, \boldsymbol{\phi})}{\max_{\boldsymbol{\theta},\boldsymbol{\phi}} \rho(\boldsymbol{O}, \boldsymbol{\varphi}|\boldsymbol{\theta}, \boldsymbol{\phi})}. \tag{10}$$

Sensitivity: $\mathrm{CR}(\boldsymbol{\mu}_{3\nu}, \boldsymbol{0}; \alpha)$ where $\boldsymbol{\mu}_{3\nu}, \boldsymbol{0}$ are mean values of $\boldsymbol{D}$&$\boldsymbol{\varphi}$.

Single-Flavor Sensitivity

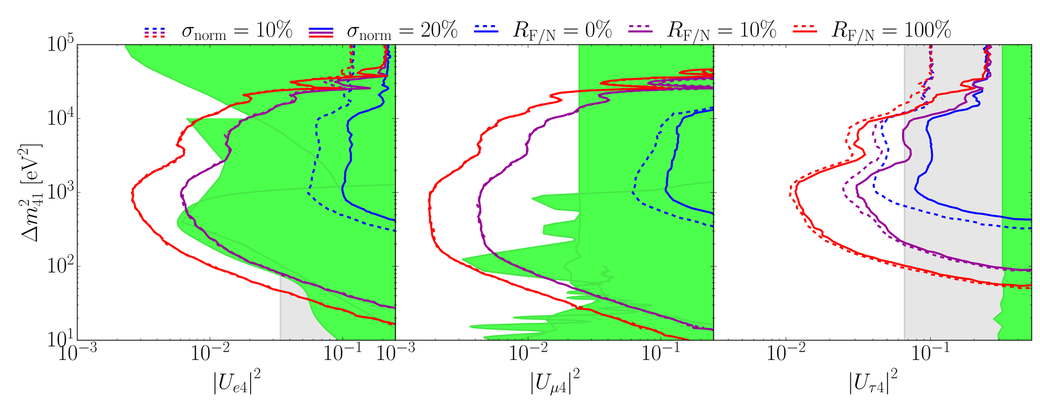

Figure 2. Sensitivities (90% CL) in the $(|U_{\alpha 4}|^2, \Delta m^2_{41})$ plane, after five years at SHiP:

- **Line styles**:
  dashed = 10% systematic uncertainty, **solid** = 20%.
- **Colors**:
  blue = NSND-only
  purple = 10% far/near ratio
  red = 100% far/near ratio.
- **Constraints**
  - green: β-decay (e), CCFR, MiniBooNE+SciBooNE & MINOS (μ) or NOvA (τ);
  - grey: Super-K (e) and IceCube-DeepCore (τ).

Adding the far detector (FSND) boosts sensitivity by up to an order of magnitude, especially near $\Delta m^2_{41} \sim 10^3\,\mathrm{eV}^2$.

Conclusion

- First full three-flavor sensitivity study of SHiP to 3+1 sterile neutrinos using Feldman–Cousins + bootstrap.
- Adding FSND at 120 m resolves appearance–disappearance cancellations and sensitivities with FSND are independent with systematic uncertainties.
- With dual baselines and 5 years at CERN SPS (2×10^{20} PoT), SHiP can reach $|U_{\alpha 4}|^2 \sim$ few $\times 10^{-3}$ near $\Delta m^2_{41} \sim 10^3\,\mathrm{eV}^2$.
- FSND increases sensitivity by up to an order of magnitude for e and μ channels (2–3 times for τ).
- Incorporates both appearance and disappearance channels across all three active flavors for comprehensive coverage.

Two-Flavor Mixing Results

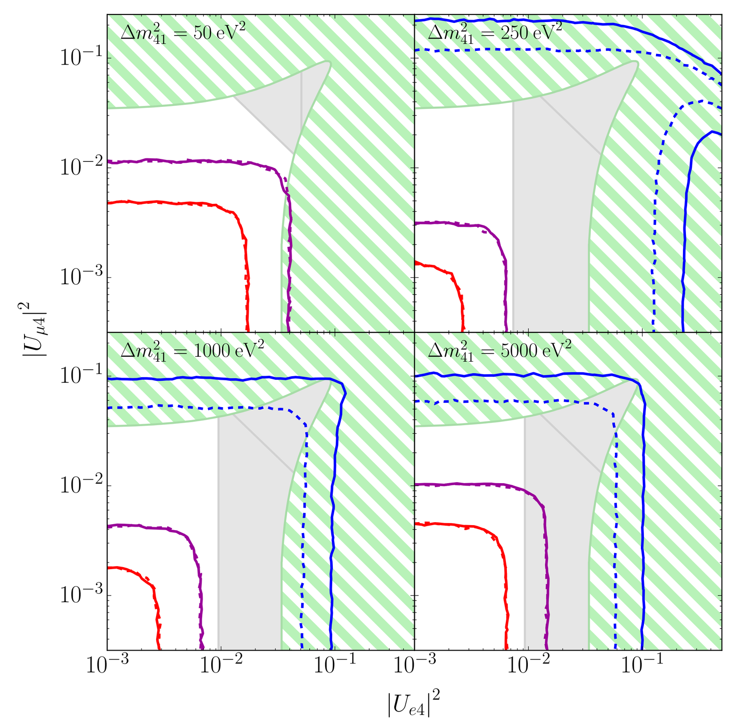

Figure 3. Sensitivities (90% CL) in the $(|U_{e4}|^2, |U_{\mu 4}|^2)$ plane

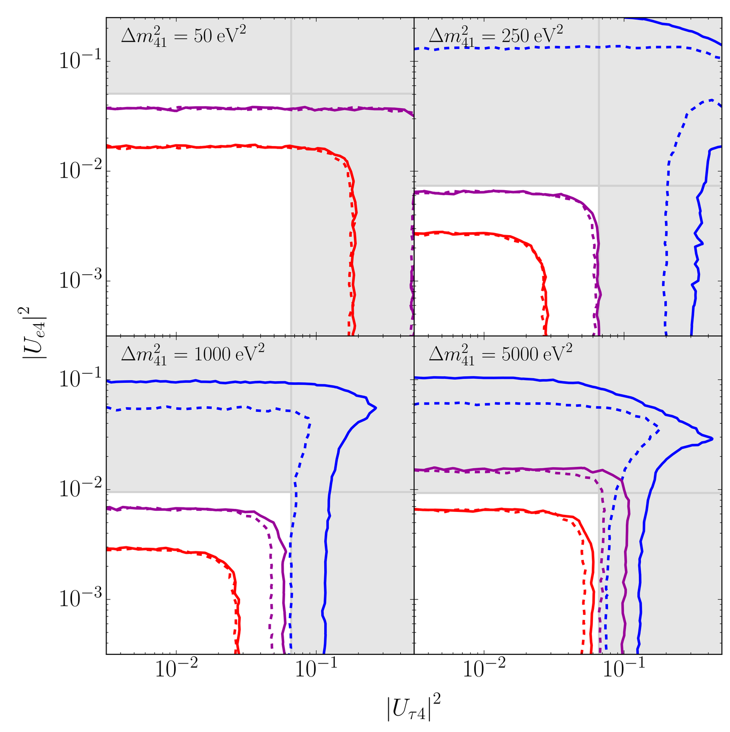

Figure 4. Sensitivities (90% CL) in the $(|U_{\tau 4}|^2, |U_{e4}|^2)$ plane

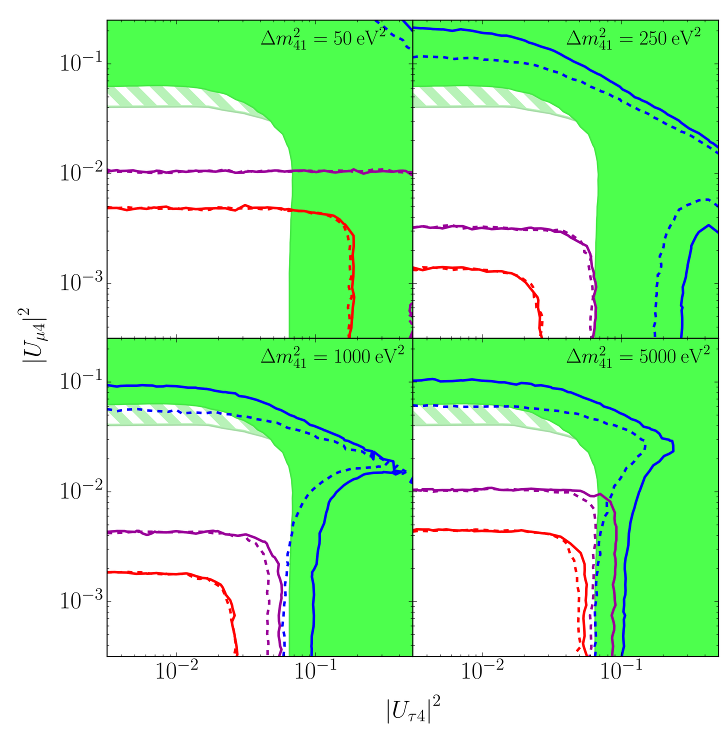

Figure 5. Sensitivities (90% CL) in the $(|U_{\tau 4}|^2, |U_{\mu 4}|^2)$ plane

- **Styling**: matches Fig. 2 (same colors & line types).
- **constraints**
  - green: IceCube-DeepCore [7] on $(|U_{\tau 4}|^2, |U_{\mu 4}|^2)$;
  - hatched light green: Super-K [8] on $(|U_{e4}|^2, |U_{\mu 4}|^2)$ and $(|U_{\tau 4}|^2, |U_{\mu 4}|^2)$ (assuming $\delta_{14} = \delta_{24} = 0$);
  - grey: tritium β-decay limits on $|U_{e4}|^2$ (vertical) and MicroBooNE appearance limits on $\sin^2 2\theta_{e\mu}$ (diagonal).

References

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