Nuclear Weak-Response Function of $^{208}{ m Pb}$ by KDAR u_{μ}

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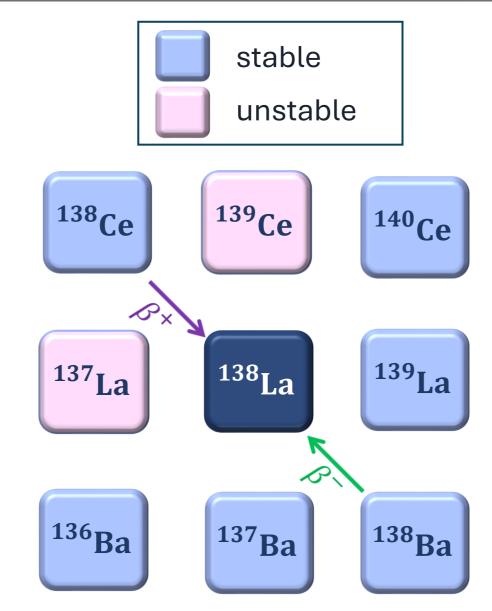
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Abstract

We investigate nuclear weak-response function of ^{208}Pb by the charged current (CC) scattering of muon neutrinos (ν_{μ}) using an incident neutrino energy of $E_{\nu_{\mu}}=236\,\text{MeV}$, as produced in kaon decay-at-rest (KDAR). In this work, we focus on the inelastic scattering occurring below the QE region. To account for these contributions, we calculate the inelastic scattering cross section using the quasiparticle random phase approximation (QRPA). In the differential cross section, we separate each multipole contribution and deduce the response functions to the KDAR probe. In particular, we compare the Gamow-Teller strength distribution in the $^{1+}$ transition to the available data with the forbidden $^{1+}$ transitions. These findings may provide valuable insight of the nuclear weak-structure comprising vector and axial currents as well as their longitudinal and transverse parts of ^{208}Pb .

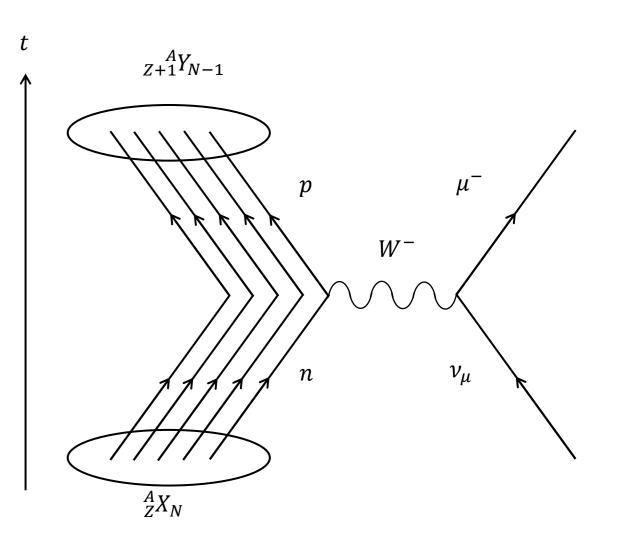
1. Introduction

- The use of KDAR neutrinos for specific nuclei, such as ²⁰⁸Pb, as in this study, can provide valuable insights into the weak structure of nuclei, including their vector and axial currents, as well as their longitudinal and transverse parts.
- KDAR neutrinos provide an optimal environment for studying neutrino-nuclear interactions because they produce a single-energy muon neutrino (vµ) beam of 236 MeV without the complexities associated with the broad energy distribution of neutrinos from meson decay-in-flight (DIF).



 138 Ba $(\nu_e, e^-)^{138}$ La

- Kaon-decay-at-rest (KDAR) muon neutrinos, with their precise 236 MeV energy, offer optimal conditions for accurately investigating neutrino-nucleus interactions and enhancing precision.
- The relatively low 236 MeV energy of KDAR muon neutrinos provides valuable opportunities for studying neutrino-nucleus interactions relevant to astrophysical phenomena such as core-collapse supernovae
- Recent experimental efforts, such as the J-PARC Spallation Neutron Source (JSNS²), actively utilize Kaon-decay-at-rest (KDAR) neutrinos to advance the study of these interactions.



2. Theoretical framework

• Quasiparticle phonon creation, annihilation operators^[1]

$$Q_{JM}^{\dagger,m} = \sum_{kl\mu'\nu'} \left[X_{(k\mu'l\nu'J)}^{m} C^{\dagger}(k\mu'l\nu'JM) - Y_{(k\mu'l\nu'J)}^{m} \tilde{C}(k\mu'l\nu'JM) \right]$$

$$Q_{JM}^{m} = \sum_{kl\mu'\nu'} \left[X_{(k\mu'l\nu'J)}^{m*} C(k\mu'l\nu'JM) - Y_{(k\mu'l\nu'J)}^{m*} \tilde{C}^{\dagger}(k\mu'l\nu'JM) \right]$$

• Differential cross section for $v(\bar{v}) - {}^{208}\text{Pb}^{[2]}$

$$\begin{split} \left(\frac{d\sigma_{\nu}}{d\Omega}\right)_{(\nu/\bar{\nu})} &= \frac{G_F^2 \epsilon k}{\pi (2J_i + 1)} \Biggl[\sum_{J=0}^{\infty} \Bigl\{ (1 + \vec{\nu} \cdot \vec{\beta}) \big| \langle J_f \| \hat{\mathcal{M}}_J \| J_i \rangle \big|^2 \\ &+ \Bigl(1 - \vec{\nu} \cdot \vec{\beta} + 2(\hat{\nu} \cdot \hat{q}) \bigl(\hat{q} \cdot \vec{\beta} \bigr) \Bigr) \Big| \langle J_f \| \hat{\mathcal{L}}_J \| J_i \rangle \Big|^2 \\ &- \widehat{q} \cdot \bigl(\hat{\nu} + \vec{\beta} \bigr) 2 \text{Re} \bigl\langle J_f \| \hat{\mathcal{L}}_J \| J_i \bigr\rangle \langle J_f \| \hat{\mathcal{M}}_J \| J_i \bigr\rangle^* \bigr\} \\ &+ \sum_{J \geq 1} \Bigl\{ \Bigl(1 - (\hat{\nu} \cdot \hat{q}) \bigl(\hat{q} \cdot \vec{\beta} \bigr) \Bigr) \Bigl(\big| \langle J_f \| \hat{\mathcal{T}}_J^{el} \| J_i \bigr\rangle \big|^2 + \big| \langle J_f \| \hat{\mathcal{T}}_J^{mag} \| J_i \bigr\rangle \big|^2 \Bigr) \\ &+ (\widehat{q} \cdot \bigl(\hat{\nu} - \vec{\beta} \bigr) 2 \text{Re} \bigl[\langle J_f \| \hat{\mathcal{T}}_J^{mag} \| J_i \bigr\rangle \langle J_f \| \hat{\mathcal{T}}_J^{el} \| J_i \bigr\rangle^* \bigr] \bigr) \Bigr\} \end{split}$$

• Coulomb, Longitudinal, Electric and Magnetic transition operators^[3]

$$\begin{split} \widehat{\mathcal{M}}_{JM;TM_{T}}(q\mathbf{x}) &= \left[F_{1}^{(T)} M_{J}^{M_{J}}(q\mathbf{x}) - i \frac{q}{M} \left[F_{A}^{(T)} \Omega_{J}^{M_{J}}(q\mathbf{x}) + \frac{F_{A} - \omega F_{P}^{(T)}}{2} \Sigma_{J}^{\prime\prime\prime M_{J}}(q\mathbf{x}) \right] \right] I_{T}^{M_{T}} \\ \widehat{\mathcal{L}}_{JM;TM_{T}}(q\mathbf{x}) &= \left[\frac{-\omega}{q} F_{1}^{(T)} M_{J}^{M_{J}}(q\mathbf{x}) + i \left(F_{A}^{(T)} - \frac{q^{2}}{2M_{N}} F_{P}^{(T)} \right) \Sigma_{J}^{\prime\prime\prime M_{J}}(q\mathbf{x}) \right] I_{T}^{M_{T}} \\ \widehat{\mathcal{T}}_{JM;TM_{T}}^{el}(q\mathbf{x}) &= \left[\frac{q}{M} \left[F_{1}^{(T)} \Delta_{J}^{\prime\prime M_{J}}(q\mathbf{x}) + \frac{1}{2} \mu^{(T)} \Sigma_{J}^{M_{J}}(q\mathbf{x}) \right] + i F_{A}^{(T)} \Sigma_{J}^{\prime\prime M_{J}}(q\mathbf{x}) \right] I_{T}^{M_{T}} \\ \widehat{\mathcal{T}}_{JM;TM_{T}}^{mag}(q\mathbf{x}) &= -i \frac{q}{M} \left[\left[F_{1}^{(T)} \Delta_{J}^{M_{J}}(q\mathbf{x}) - \frac{1}{2} \mu^{(T)} \Sigma_{J}^{\prime\prime M_{J}}(q\mathbf{x}) \right] + F_{A}^{(T)} \Sigma_{J}^{M_{J}}(q\mathbf{x}) \right] I_{T}^{M_{T}} \end{split}$$

• The 7 relevant single particle operators^[3]

$$\begin{split} M_{J}^{M} & \Delta_{J}^{MJ} = \mathbf{M}_{JJ}^{MJ}(q\mathbf{x}) \cdot \frac{1}{q} \nabla \qquad \Sigma_{J}^{MJ} = \mathbf{M}_{JJ}^{MJ}(q\mathbf{x}) \cdot \sigma \\ \Delta_{J}^{\prime MJ} = [J]^{-1} \left[-J^{\frac{1}{2}} \mathbf{M}_{JJ+1}^{MJ}(q\mathbf{x}) + (J+1)^{\frac{1}{2}} \mathbf{M}_{JJ-1}^{MJ}(q\mathbf{x}) \right] \cdot \frac{1}{q} \nabla \\ \Sigma_{J}^{\prime MJ} = [J]^{-1} \left[-J^{\frac{1}{2}} \mathbf{M}_{JJ+1}^{MJ}(q\mathbf{x}) + (J+1)^{\frac{1}{2}} \mathbf{M}_{JJ-1}^{MJ}(q\mathbf{x}) \right] \cdot \sigma \\ \Sigma_{J}^{\prime \prime MJ} = [J]^{-1} \left[-(J+1)^{\frac{1}{2}} \mathbf{M}_{JJ+1}^{MJ}(q\mathbf{x}) + J^{\frac{1}{2}} \mathbf{M}_{JJ-1}^{MJ}(q\mathbf{x}) \right] \cdot \sigma \\ \Omega_{J}^{MJ}(q\mathbf{x}) = M_{J}^{MJ}(q\mathbf{x}) \sigma \cdot \frac{1}{q} \nabla \qquad \Omega_{J}^{\prime MJ}(q\mathbf{x}) = \Omega_{J}^{MJ}(q\mathbf{x}) + \frac{1}{2} \Sigma_{J}^{\prime \prime MJ}(q\mathbf{x}) \end{split}$$

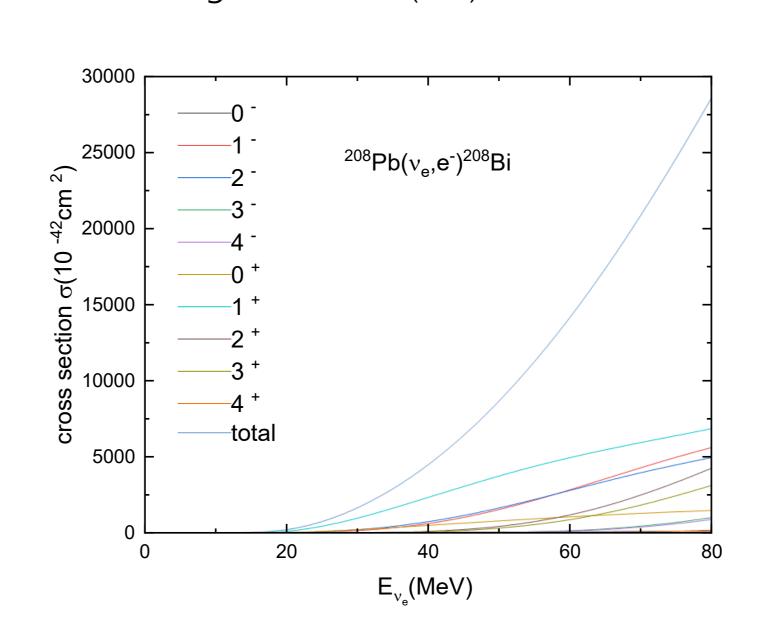
[1] J. Suhonen, *From Nucleons to Nucleus*, (Springer, Berlin, 2007)

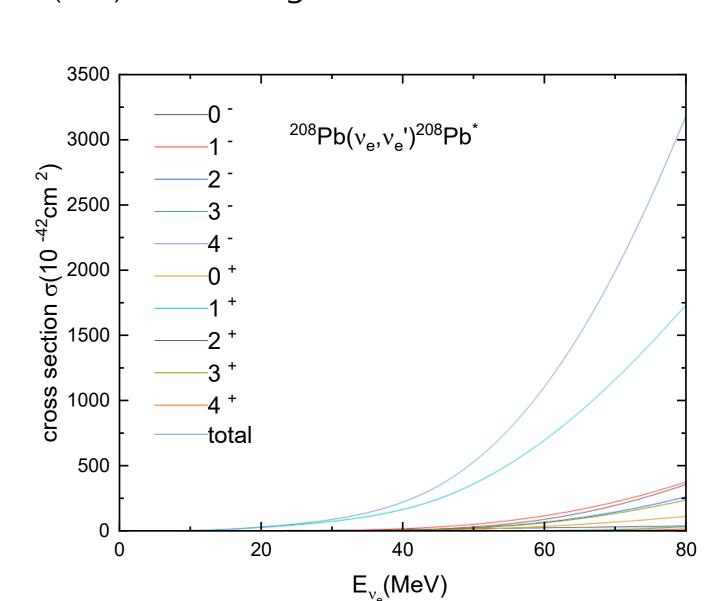
[2] J. D. Walecka, Muon Physics, edited by V. H. Huges and C. S. Wu (Academic, New York, 1975), Vol II.

[3] T. W. Donnelly and W. C. Haxton, ATOMIC DATA AND NUCLEAR DATA 23, 103 (1979).

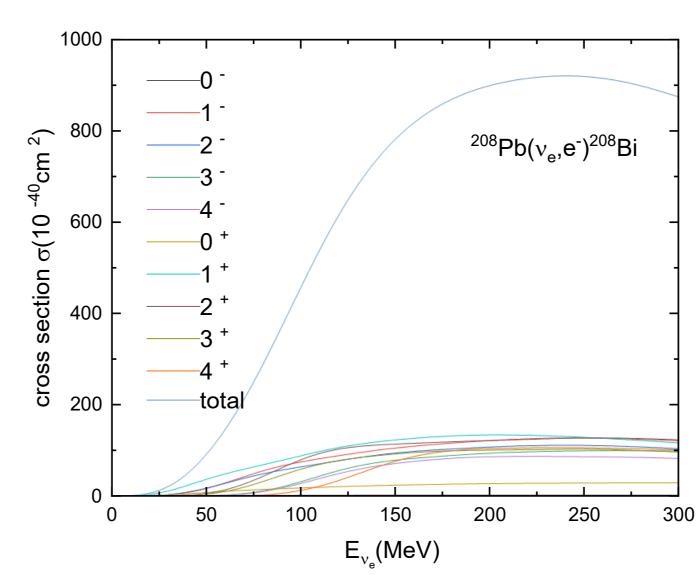
3. Results and Discussions

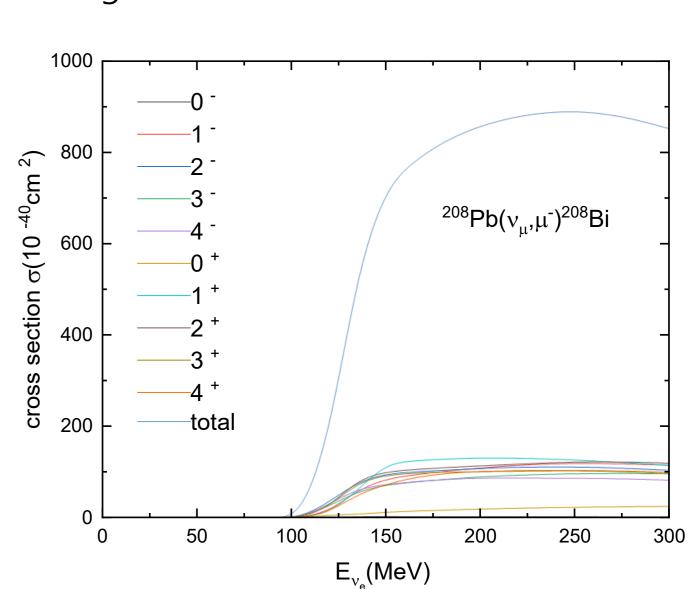
Charged current(CC) and neutral current(NC) scattering cross sections





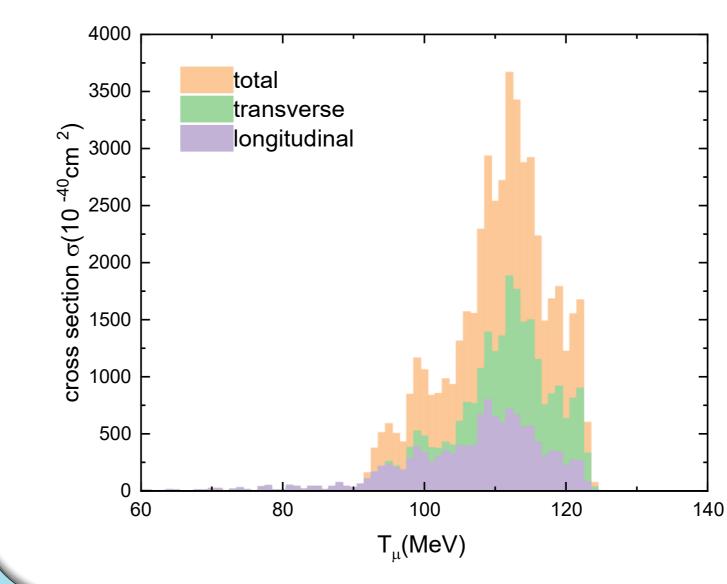
Electro neutrino and muon neutrino scattering cross sections

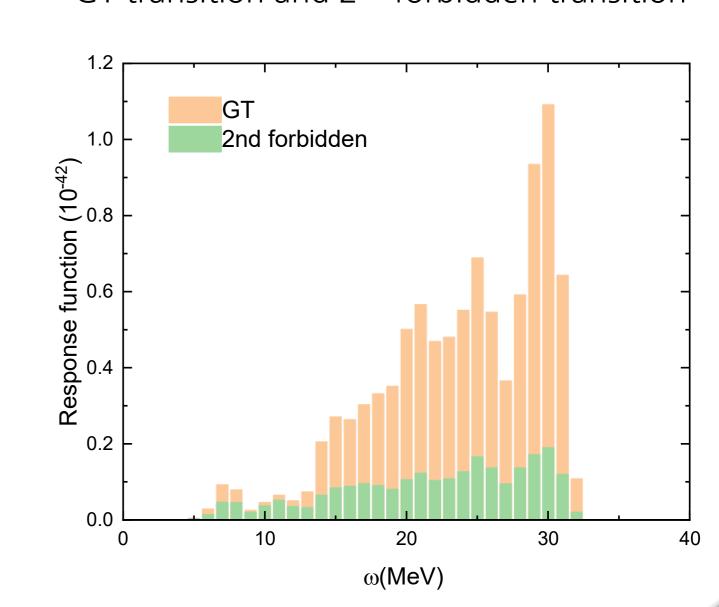




The contribution from electro-magnetic transverse part







4. Summary

- This study developed and validated a calculational methodology for KDAR muon neutrino scattering off ²⁰⁸Pb, combining RMF/QMC (DWBA) for quasielastic and QRPA for inelastic processes.
- KDAR muon neutrino scattering exhibits dominant inelastic contributions above 80 MeV muon kinetic energy, which are accurately modeled by QRPA, providing critical insights into astrophysical phenomena like core-collapse supernovae.
- By disentangling multipole contributions from differential cross sections, this research derived the ²⁰⁸Pb nuclear weak response function, crucial for understanding its weak structure and awaiting validation from future low-energy neutrino experiments (e.g., J-PARC, DUNE) for fundamental neutrino physics