$e-\mu$ collider to discover heavy sterile neutrinos

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Future muon collider

Why muon collider:

1) Muon produced in high energy collisions

naturally by collision of cosmic rays to Earth atmosphere,

artificially by particle accelerators.

2) Clean environment as

elementary particle, unlike proton collider (LHC),

no strong interaction (as lepton)

3) Easy to accelerate to higher energy than electron

less synchrotron radiation in (high energy) future circular collider,

rather long lifetime, compared to other elementary particles

$$\tau = \gamma \times \tau_0 = \frac{1}{\sqrt{1-(v/c)^2}} \times (2.2 \mu s)$$

Future muon collider

Synchrotron radiation: Power carried by the radiation (relativistic Larmor formula)

Cyclotron radiation
Synchrotron radiation

$$P_\gamma = \frac{q^2}{6\pi\varepsilon_0 c^3} a^2 \gamma^4 = \frac{q^2 c}{6\pi\varepsilon_0} \frac{\beta^4\gamma^4}{\rho^2},$$

where

- ε_0 is the vacuum permittivity,
- q is the particle charge,
- a is the magnitude of the acceleration,
- c is the speed of light,
- γ is the Lorentz factor,
- $\beta = v/c$,
- ρ is the radius of curvature of the particle trajectory.

If $E(e) = E(\mu)$, $\gamma(e) \approx 210 \times \gamma(\mu)$

Future electron-muon collider

Why e-mu collider:

1) observing cLFV (at high energy), which is beyond the SM.

$$e^{\pm}\mu^{\mp} \to f_1 f_2 \qquad f_{1,2} = \text{lepton, quark, gauge boson, bSM particle}$$

2) Comparing w/ usual cLFV experiments.

$e\mu \to f_1 f_2$ vs. $\mu \to e\gamma$ High scale vs. Low scale

$e\mu \to f_1 f_2$ vs. $\mu \to eee$ High scale vs. Low scale

$e\mu \to f_1 f_2$ vs. $ee \to e\mu$ Many process vs. Single process

3) We will consider $e\mu \to W^+W^-$ $e^{\pm}\mu^{\mp} \to e^{\mp}\mu^{\pm}$

charged Lepton Flavor Violation (cLFV)

lepton flavor number:

$L_e = 1:\ e^-\quad L_e = -1:\ e^+\qquad L_\mu = 1:\ \mu^-\quad L_e = -1:\ \mu^+\qquad L_\tau = 1:\ \tau^-\quad L_\tau = -1:\ \tau^+$

lepton number:

$L = L_e + L_\mu + L_\tau$

➔ SM conserves (total) lepton number ➔ LNC and cLFC

$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$

➔ neutrino oscillation, $\nu_e \leftrightarrow \nu_\mu \leftrightarrow \nu_\tau$, violate ➔ (neutral) LFV

➔ cLFV only bSM, $\mu^- \to e^- + \gamma$ $e\mu \to W^+W^-$ $e^\pm\mu^\mp \to e^\mp\mu^\pm$

Why heavy sterile neutrinos ?

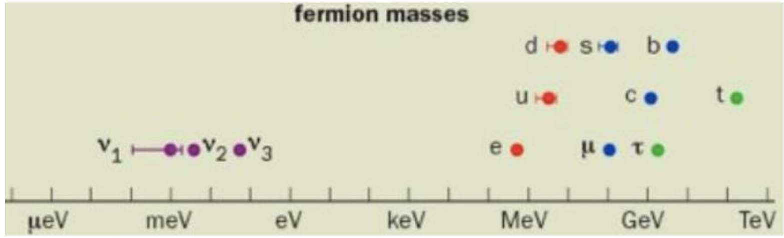
Neutrinos in (minimal) SM massless $m_\nu = 0$ ← only left-handed ν_L

$$\mathscr{L}^D_{\rm mass} = -m_\nu \left(\overline{\nu_R}\nu_L + \overline{\nu_L}\nu_R\right)$$

However, neutrinos have mass $m_\nu \neq 0$ ← neutrino oscillation

→ existence of (sterile) right-handed AND/OR sub-eV active neutrinos are Majorana

Neutrinos in SM have tiny-tiny mass

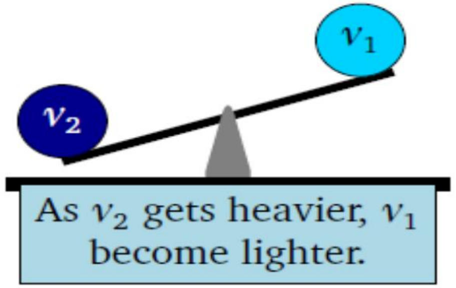


→ possibly existence suppressive high scale

seesaw mechanism with massive right-handed neutrino

→→ **heavy sterile (Majorana) neutrino**

possible LNV observable



$$\Longrightarrow m_1 \approx -\frac{\left(m_\nu^D\right)^2}{m_\nu^R} \text{ and } m_2 \approx m_\nu^R. \qquad \sin\theta \approx m_\nu^D / m_\nu^R$$

Major unresolved problems in SM – requiring BSM

Neutrino oscillation and (tiny) masses → heavy sterile (Majorana) neutrino w/ seesaw

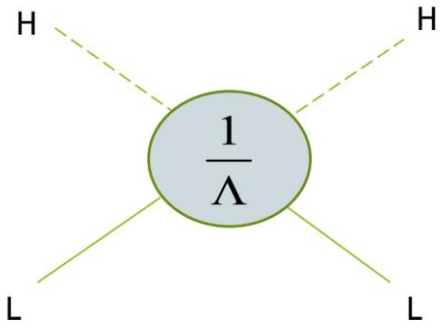
Dark mater (DM) → *renormalizable* BSM w/ heavy and/or weakly interacting particles

vector portal, scalar portal → Neutrino portal w/

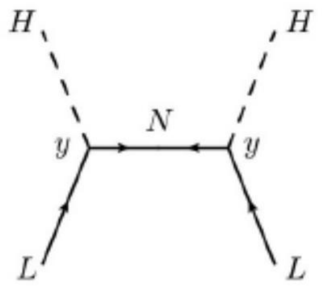
$$\mathcal{L}_{\text{Neutrino portal}} = F_{\alpha I}(\bar{L}_\alpha \cdot \tilde{\Phi})N_I.$$ → require right-handed (sterile) neutrino N

$$+ M_I \bar{N}_I^C N_I$$ ← if Majorana particle

$$\mathcal{L}_{5-dim} \sim \frac{1}{\Lambda}(L_i H)^T H L_j$$ ← at scale < m(N), as Weinberg operator



Type-I

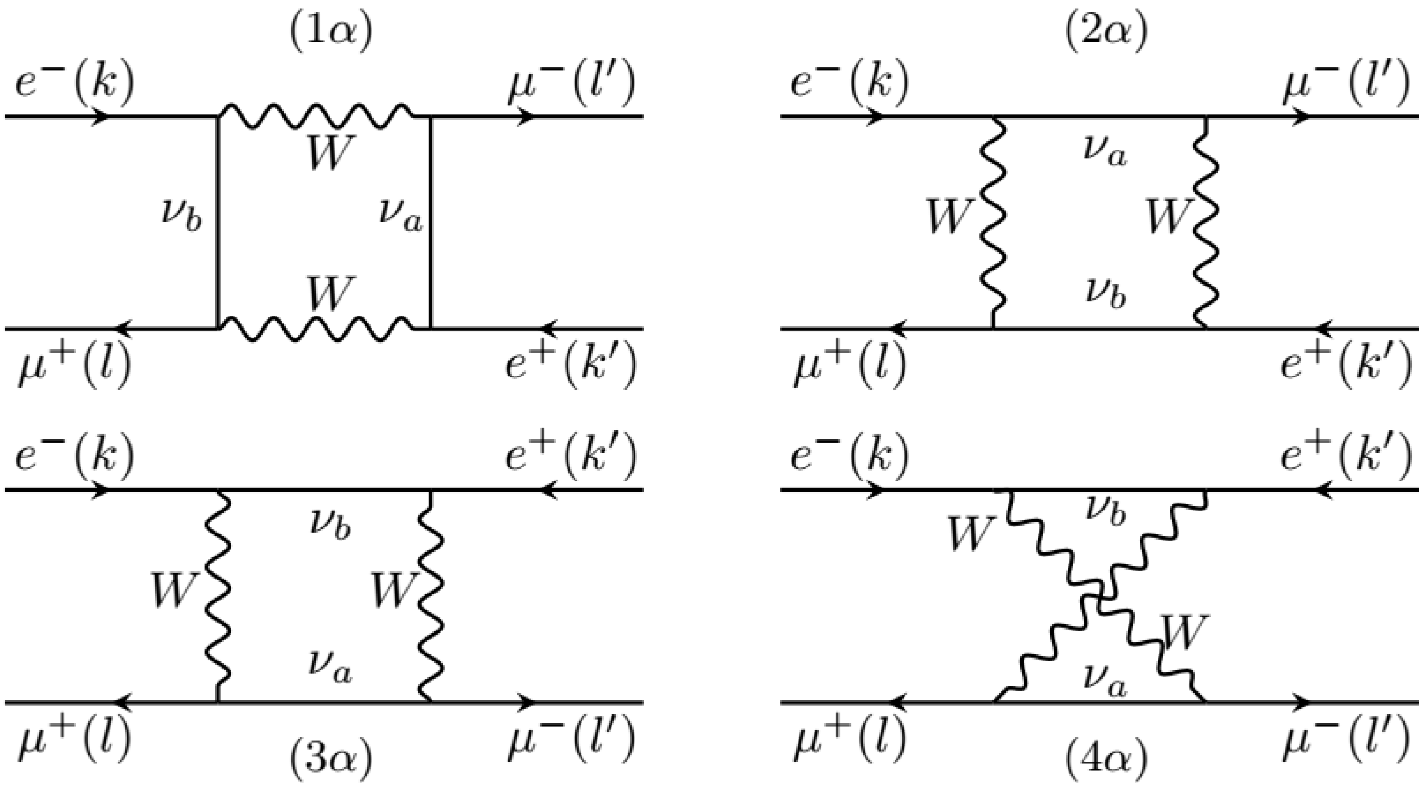


$$m_\nu \propto \frac{v^2 y^2}{M_N}$$

→ <u>heavy sterile N = DM</u>

most imminent minimal extension of the SM

$e^- + \mu^+ \to e^+ + \mu^-$ for $\sqrt{s} < 2m_W$



$$\nu_{a,b} = \nu_1, \nu_2, \nu_3, N_s (= N_1, N_2)$$

If only 3 nu as SM, unitarity of PMNS gives

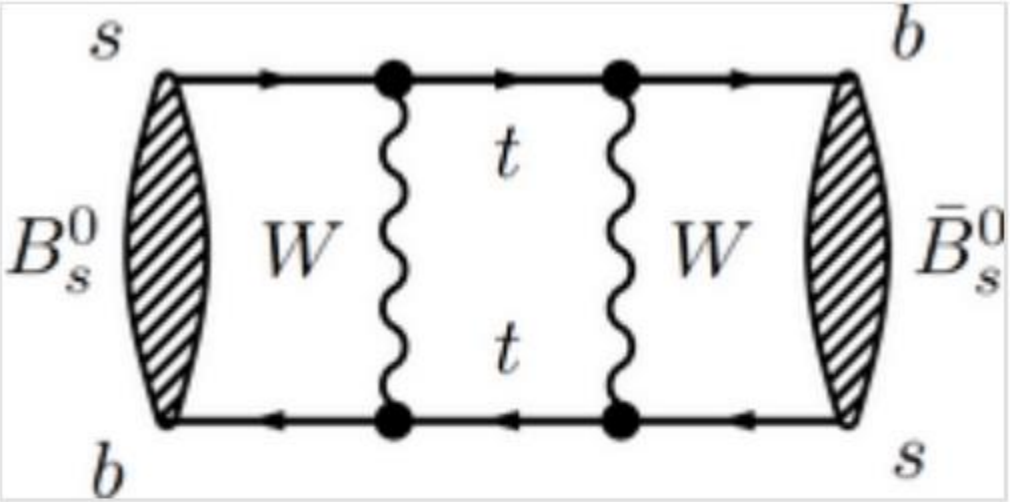
$$\sum_{a=1}^{3} U^*_{ea} U_{\mu a} = 0 \quad \rightarrow \quad \sum Box = 0$$

Box diagrams w/ large s=(k+l)^2

Inami-Lim function → generalized Inami-Lim

FIG. 1. The box diagrams for the scattering process $e^-(k)\mu^+(\ell) \to \mu^-(\ell')e^+(k')$, of the type $(j\alpha)$ $(j = 1, 2, 3, 4)$, where α indicates that these are the box diagrams with two W-propagators. The boxes also contain two neutrino propagators (of neutrinos ν_a and ν_b that can be light or heavy). The diagrams of the type (jk) with $k = \beta, \gamma, \delta$ (and $j = 1, 2, 3, 4$) are those where one or both W-propagators are replaced by the Goldstone propagator (see the text).

$B-\bar{B}$ Mixing (Oscillation) and Box Diagram



The oscillations are mediated by the W boson and the top quark; being heavier than the B_s–$\bar{B}_s$ mesons, they act as short lived virtual particles

$$F_2(x_a,x_b) \equiv i16\pi^2 M_W^2 \int \frac{d^4p}{(2\pi)^4} \frac{p^2}{(p^2-M_a^2)(p^2-M_b^2)(p^2-M_W^2)^2}$$
$$= \frac{1}{(x_a-x_b)}\left[\frac{1}{(1-x_a)} + \frac{x_a^2}{(1-x_a^2)}\ln x_a - (x_a \leftrightarrow x_b)\right]$$
$$F_0(x_a,x_b) \equiv i16\pi^2 M_W^2 \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2-M_a^2)(p^2-M_b^2)(p^2-M_W^2)^2}$$
$$= \frac{1}{(x_a-x_b)}\left[\frac{1}{(1-x_a)} + \frac{x_a}{(1-x_a^2)}\ln x_a - (x_a \leftrightarrow x_b)\right].$$

Inami-Lim Function, $F_{0,2}(x_i = \frac{m_i^2}{m_W^2})$, i=u,c,t

ARGUS exp. (1987) observed large mixing,
predicted large top-quark (>100 GeV) mass.

$e^- + \mu^+ \to e^+ + \mu^-$ for $\sqrt{s} < 2m_W$

$$A = (g/\sqrt{2})^4 \mathcal{A}_{\rm box}, \qquad \mathcal{A}_{\rm box} = \sum \mathcal{F}_f^{(jk)}, \text{ where } j = 1,2,3,4 \text{ and } k = \alpha, \beta, \gamma, \delta$$

$$\mathcal{F}^{(1\alpha)} = \int \frac{d^4p}{(2\pi)^4} \frac{(-1)\left[\overline{u}(\ell')\gamma^\alpha P_L(\not{p} + \not{\ell}' + M_a)\gamma^\beta P_L v(k')\right]\left[\overline{v}(\ell)\gamma_\beta P_L(\not{p} + \not{k} + M_b)\gamma_\alpha P_L u(k)\right]}{((p+\ell')^2 - M_a^2 + i\Gamma_a M_a)((p+k)^2 - M_b^2 + i\Gamma_b M_b)((p+k+\ell)^2 - M_W^2 + i\Gamma_W M_W)(p^2 - M_W^2 + i\Gamma_W M_W)}$$

$$\sum_h u(k)\overline{u}(k) = (\not{k} + m_e);\ (k+\ell)^2 = s;\ (k-k')^2 = t;\ (k-\ell')^2 = u. \qquad m_e, m_\mu \mapsto 0. \quad s + t + u = 0.$$

$$\mathcal{F}^{(1\alpha)} = \frac{i}{16\pi^2}\frac{2}{M_W^2}\left\{\overline{F}_1^{(1)}\left[\overline{u}(\ell')\gamma^\mu P_L v(k')\right]\left[\overline{v}(\ell)\gamma_\mu P_R u(k)\right] - 2\overline{F}_2^{(1)}(1-x, 1-y)\frac{1}{M_W^2}\left[\overline{u}(\ell')\not{k} P_L v(k')\right]\left[\overline{v}(\ell)\not{\ell}' P_L u(k)\right]\right\}$$

$$\overline{F}_1^{(1)} = \int dxdydz \frac{1}{\Delta_W^{(1)}}, \quad \overline{F}_2^{(1)}(f_1, f_2) = \int dxdydz \frac{f_1 f_2}{\left(\Delta_W^{(1)}\right)^2}, \qquad 0 \le x+y+z \le 1 \text{ and } 0 \le x,y,z \le 1$$

Feynman-Schwinger parameters

$$\Delta_W^{(1)} = \left[(1-x-y)(1-s_W z) + s_W z^2 + xy(-u_W) + x x_a + y x_b\right] - i\varepsilon_W - i\varepsilon_N$$

$$s_W = \frac{s}{M_W^2},\ u_W = \frac{u}{M_W^2},\ t_W = \frac{t}{M_W^2},$$
$$x_a = \frac{M_a^2}{M_W^2},\ x_b = \frac{M_b^2}{M_W^2},\ x_W \equiv M_N^2/M_W^2,$$
$$\varepsilon_W = \frac{\Gamma_W}{M_W}(1-x-y),\ \varepsilon_N = x\frac{\Gamma_a M_a}{M_W^2} + y\frac{\Gamma_b M_b}{M_W^2}.$$

$e^- + \mu^+ \to e^+ + \mu^-$ for $\sqrt{s} < 2m_W$

$$A = (g/\sqrt{2})^4 \mathcal{A}_{\rm box}, \qquad \mathcal{A}_{\rm box} = \sum \mathcal{F}_f^{(jk)}, \text{ where } j = 1,2,3,4 \text{ and } k = \alpha,\beta,\gamma,\delta$$

Now considering PMNS & heavy-light mixings:

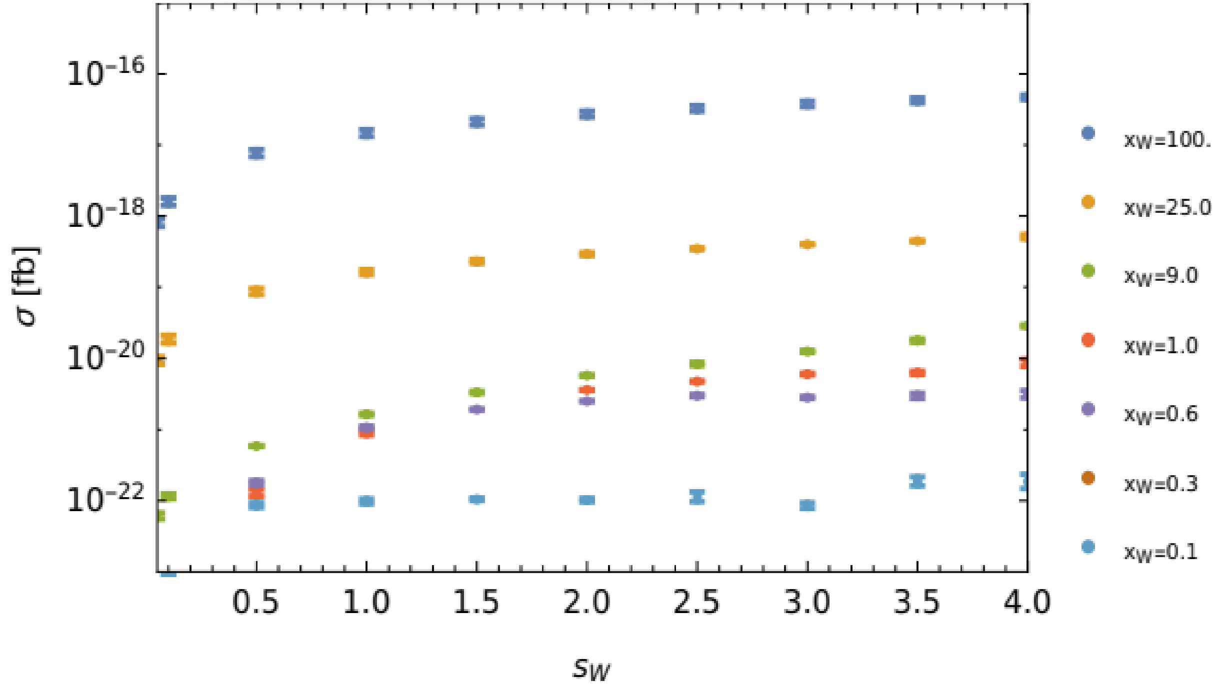
neutrino eigenstates ν_η $(\eta = e, \mu, \tau)$

$$\nu_\eta = \sum_{a=1}^{3+N_H} U_{\eta a}\nu_a = \sum_{a=1}^{3} U_{\eta a}\nu_a + \sum_{b=1}^{N_H} U_{\eta N_b} N_b,$$

Due to Unitarity of neutrino mixing matrix

$$\sum_{a=1}^{3} U_{ea}^* U_{\mu a} = -U_{eN}^* U_{\mu N}$$

$$\sigma(e^-\mu^+ \to \mu^- e^+) = \frac{1}{32\pi s}\left(\frac{g}{\sqrt{2}}\right)^4 \int_{-1}^{+1} d(\cos\theta) \left\langle \left| \sum_{j,k} \mathcal{F}_f^{(jk)} \right|^2 \right\rangle$$



$$x_W = \frac{m_N^2}{m_W^2}$$

$$s_W = \frac{s}{m_W^2}$$

Assumed $|(U_{\mu N} U_{eN}^*)^2| = 10^{-10}$

Heavy-light mixing (in type I seesaw)

In type-I seesaw, SM + heavy Majorana neutrinos N_R (n_H=2).
N_R couples to SM neutrinos and Higgs via Yukawa couplings Y_nu, and generate Dim=5 Weinberg operator at low scale.
After e-w symmetry breaking, Weinberg operator give tiny Majorana masses to active SM neutrinos,

$$M_\nu = -m_D D_N^{-1} m_D^T \qquad m_D = Y_\nu v/\sqrt{2} \ll D_N \qquad D_N = \mathrm{diag}(M_{N_1}, M_{N_2})$$

Now adopting Casa-Ibarra parametrization (to ensure PMNS and neutrino masses consistent to oscillation data) :

$$m_D = U_\nu \sqrt{D_\nu} R \sqrt{D_N},$$

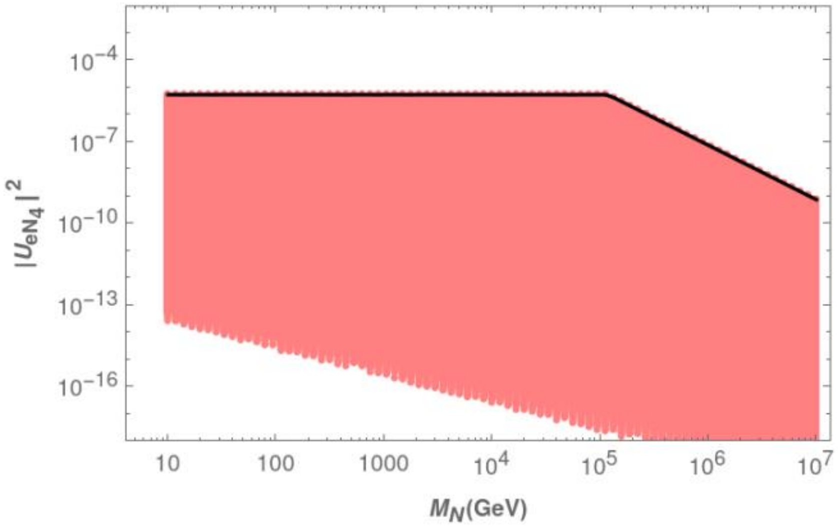
U_ν is the 3 × 3 PMNS $\quad$ R is a 3 × 2 complex orthogonal matrix

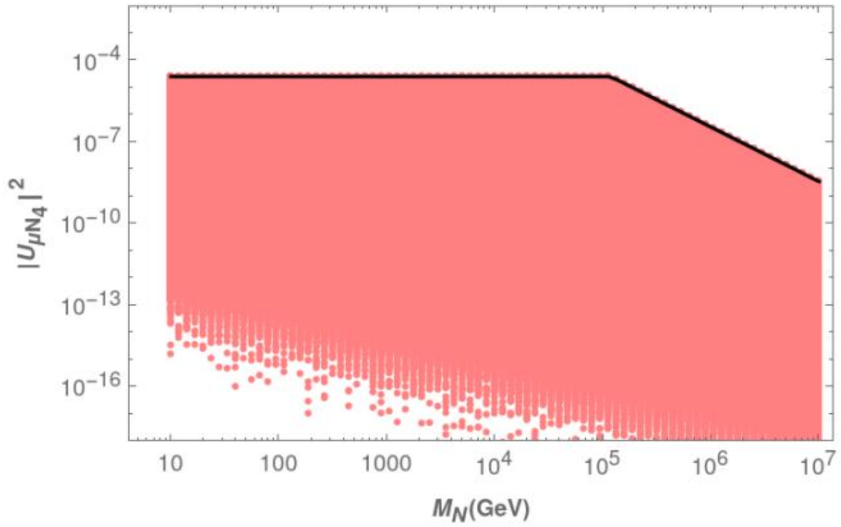
$D_\nu = \mathrm{diag}(M_1, M_2, M_3)$ is the diagonal matrix of light neutrino

$$D_\nu = \mathrm{diag}\left(0,\ \sqrt{\Delta m^2_{\rm sol}},\ \sqrt{\Delta m^2_{\rm sol} + \Delta m^2_{\rm atm}}\right)$$

Then, we get $\quad U_{\alpha N_{4,5}} = m_D D_N^{-1}$

So we assumed $\quad |(U_{\mu N} U^*_{eN})^2| = 10^{-10}$



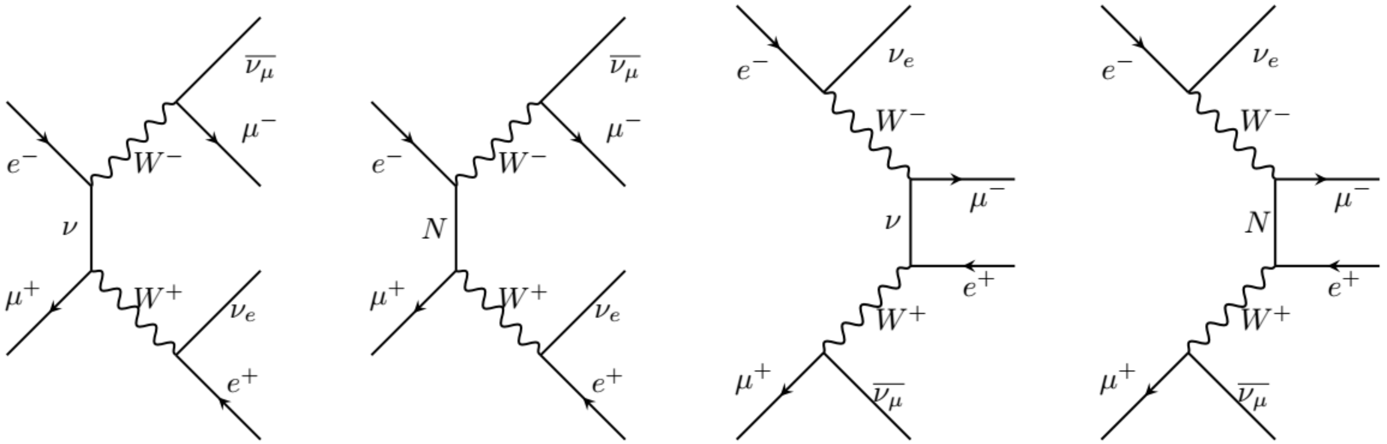


$e^- + \mu^+ \to W^+ + W^-$ for $\sqrt{s} > 2m_W$

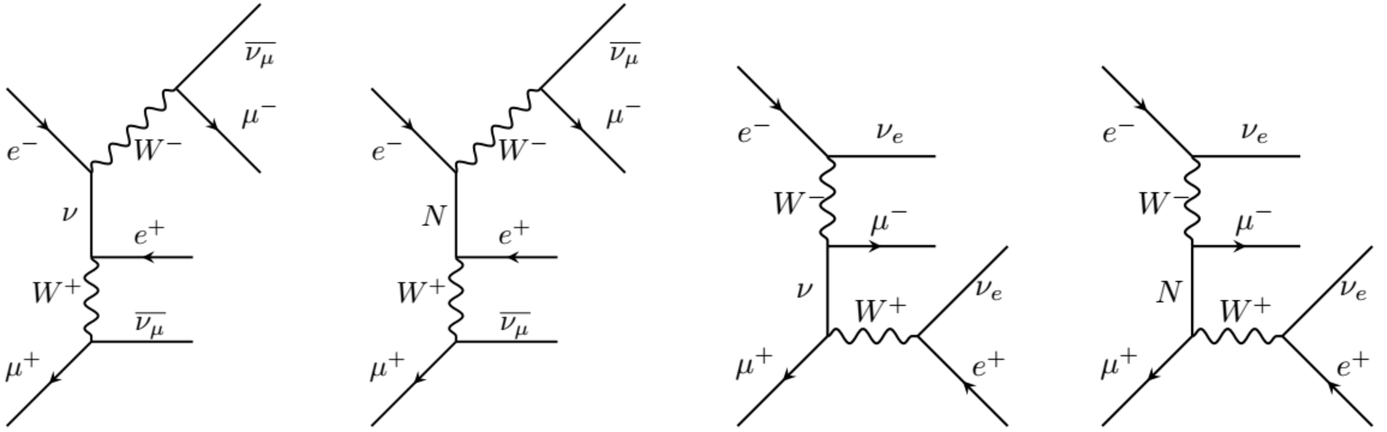
If we consider for $\sqrt{s} < 2m_W$

$$e^- \mu^+ \to \mu^- e^+ + 2\,\nu$$

LFV ➔



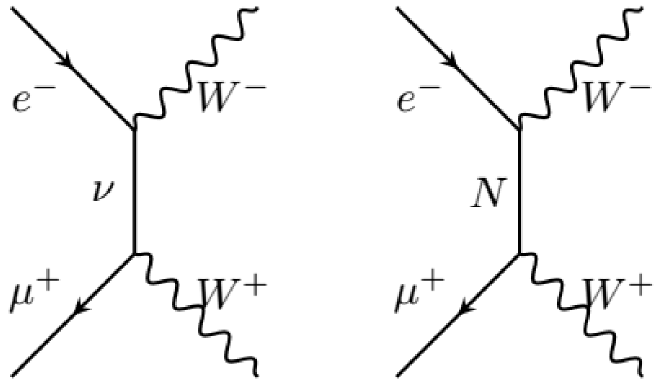
LNV + LFV ➔



W/o observing final nu, no way to distinguish LNV/LFV, neither intermediate nu from nu or N

$e^- + \mu^+ \rightarrow W^+ + W^-$ for $\sqrt{s} > 2m_W$

Instead, we observe on-shell W+, W-, which decays leptonically or hadronically.



$$\sigma(s) = \frac{1}{16\pi s}\int_{t_-}^{t_+} |\mathcal{M}|^2\, dt,$$

$$|\mathcal{M}|^2 = \frac{g^4}{16M_W^4}\left(-4M_W^8 + 8M_W^6 t - 4M_W^4 st - 5M_W^4 t^2 + 4M_W^2 st^2 + 2M_W^2 t^3 - st^3 - t^4\right)\left|\sum_{a=1}^{3+N_H}\frac{U_{ea}^* U_{\mu a}}{t - M_a^2}\right|^2 .$$

$$t_\pm = M_W^2 - \frac{s}{2}(1 \mp \beta),$$

$$\beta = \sqrt{1 - \frac{4M_W^2}{s}}.$$

$e^- + \mu^+ \rightarrow W^+ + W^-$ for $\sqrt{s} > 2m_W$

We get very NICE relation, due to Unitarity of Neutrino Mixing Matrix.

Considering only 3 light nu ➔

$$\sum_{a=1}^{3} \frac{U_{ea}^* U_{\mu a}}{t - M_a^2} \xrightarrow{M_a^2=0} \frac{1}{t} \sum_{a=1}^{3} U_{ea}^* U_{\mu a} = 0,$$

Due to Unitarity of 3X3 PMNS
➔ if only 3 nu of SM ➔ 0

Considering 3 light nu + heavy N ➔

$$\sum_{a=1}^{n} \frac{U_{ea}^* U_{\mu a}}{t - M_a^2} = \frac{1}{t} \sum_{a=1}^{3} U_{ea}^* U_{\mu a} + \sum_{a=4}^{N_H} \frac{U_{eN_a}^* U_{\mu N_a}}{t - M_a^2},$$

Due to Unitarity

$$\sum_{a=1}^{3} U_{ea}^* U_{\mu a} = -U_{eN}^* U_{\mu N}$$

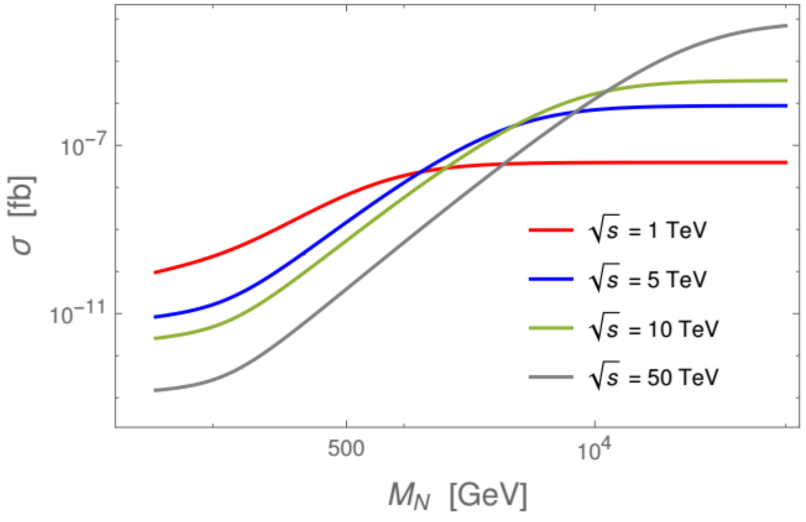
With one heavy N ➔

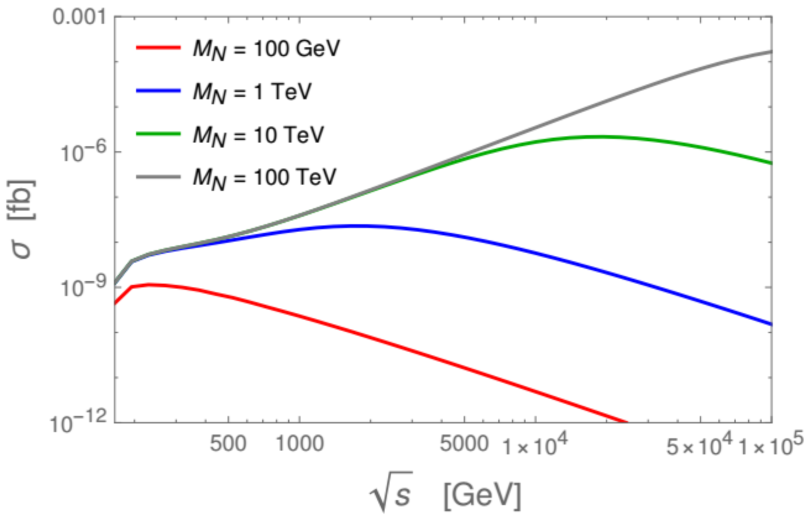
$$\sum_{a=1}^{4} \frac{U_{ea}^* U_{\mu a}}{t - M_a^2} = -\frac{1}{t} U_{eN}^* U_{\mu N} + \frac{U_{eN}^* U_{\mu N}}{t - M_N^2}.$$

$$|U_{eN}^* U_{\mu N}| = 10^{-5}$$

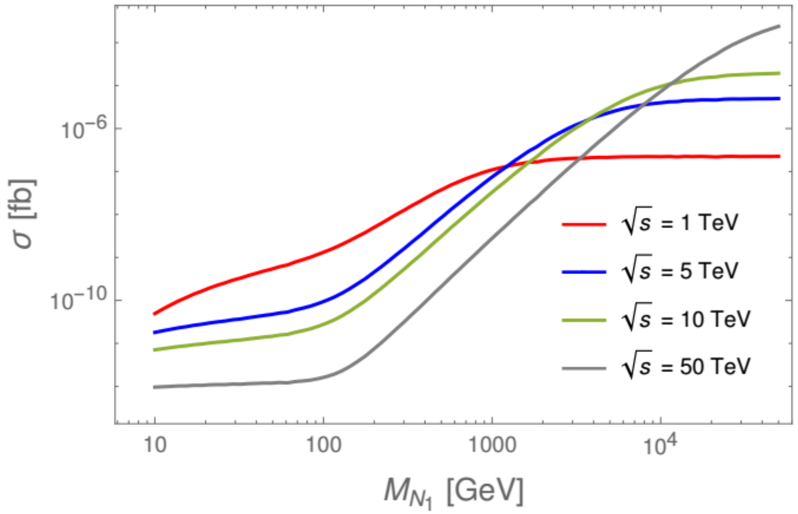
$e^- + \mu^+ \to W^+ + W^-$ for $\sqrt{s} > 2m_W$

With one heavy N ➔

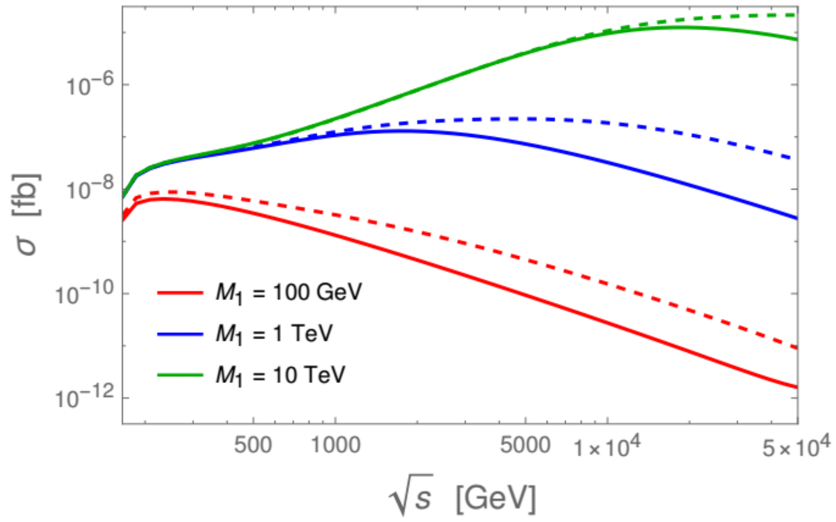




With 2 heavy N (type 1 seesaw) & M(N1)=M(N2)=M_N ➔



M(N2)=5M(N1) ➔



Summary

1. e-mu collider, as application of possible future mu-mu collider, is the best place to test charged Lepton Flavor Violation (cLFV), and to discover "heavy sterile (Majorana?) neutrino (HSN)".

2. HSN can be a candidate of DM, can give light neutrinos masses, tiny-tiny mass through (type 1) seesaw mechanism.

3. We investigated the discovery limit of heavy sterile neutrino via $e^- + \mu^+ \to e^+ + \mu^-$ for $\sqrt{s} < 2m_W$ and $e^- + \mu^+ \to W^+ + W^-$ for $\sqrt{s} > 2m_W$.

감사(感謝, thanks)

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