Multipole-Resolved Nuclear Transition Strengths of $^{12}\mathrm{C}$ Induced by Mono-Energetic KDAR Neutrinos

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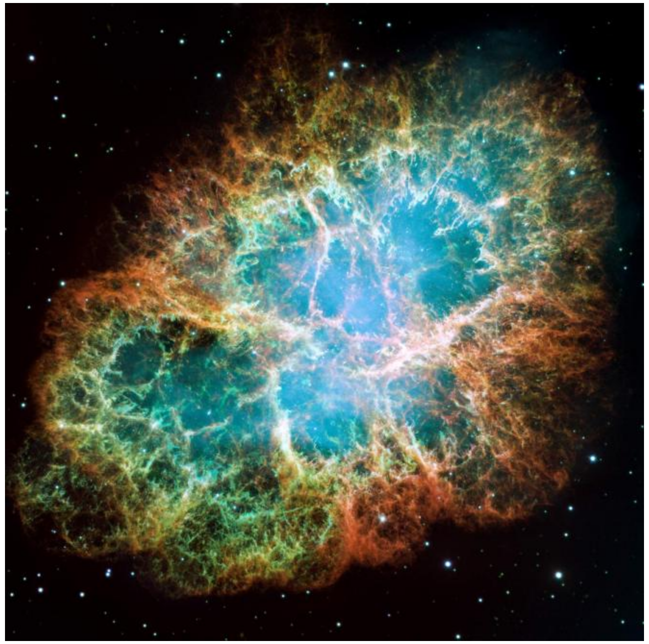
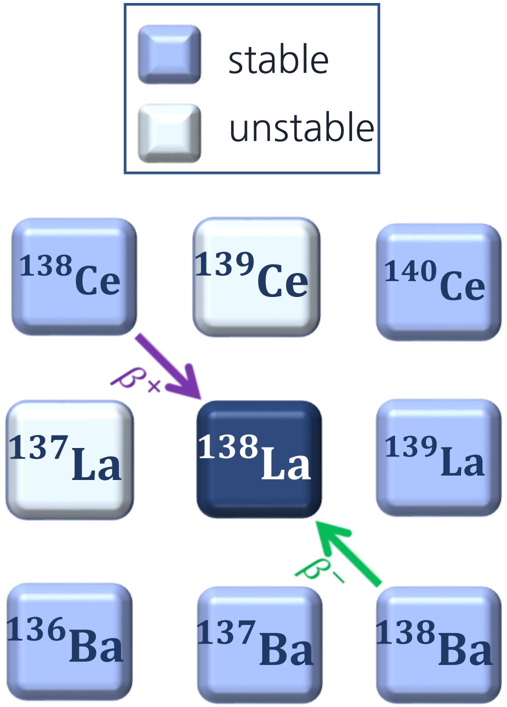
Origin of Matter and Evolution of Galaxies

Introduction



[1] https://www.nasa.gov/sites/default/files/thumbnails/image/crab-nebula-mosaic.jpg

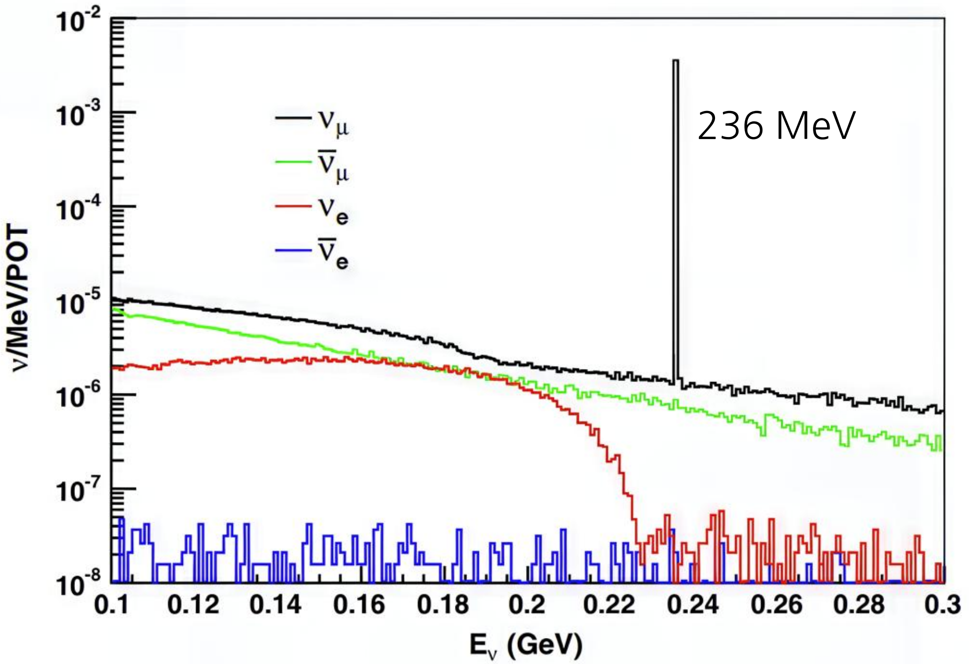
- Neutrino-induced reactions and neutrino scattering on complex nuclei plays an important role understanding of nuclear structure as probed by the weak interaction.
- Neutrino-induced reactions have been known to play **important roles on the nucleosynthesis** in core collapsing supernovae explosions.
- For heavy nuclei, certain nuclei that can not be produced by beta decay because of surrounding stable nuclei are produced by the neutrino process ($\nu - \text{process}$).
- High energy neutrinos from supernovae interact with nuclei serve as weak fast process in network calculations that estimate the abundance of nuclei.



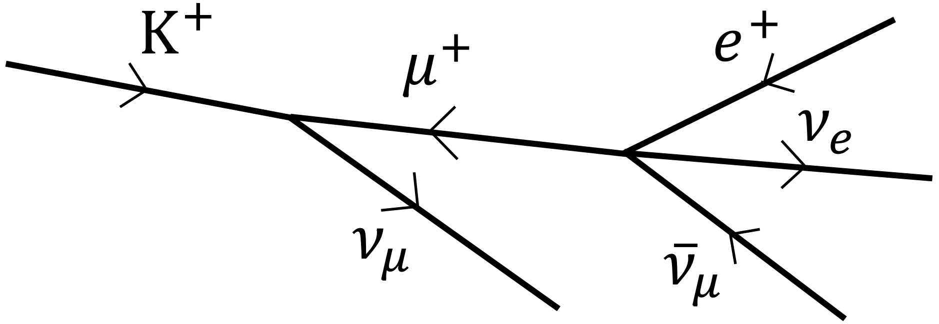
$$^{138}\text{Ba}(\nu_e, e^-)^{138}\text{La}$$

Introduction

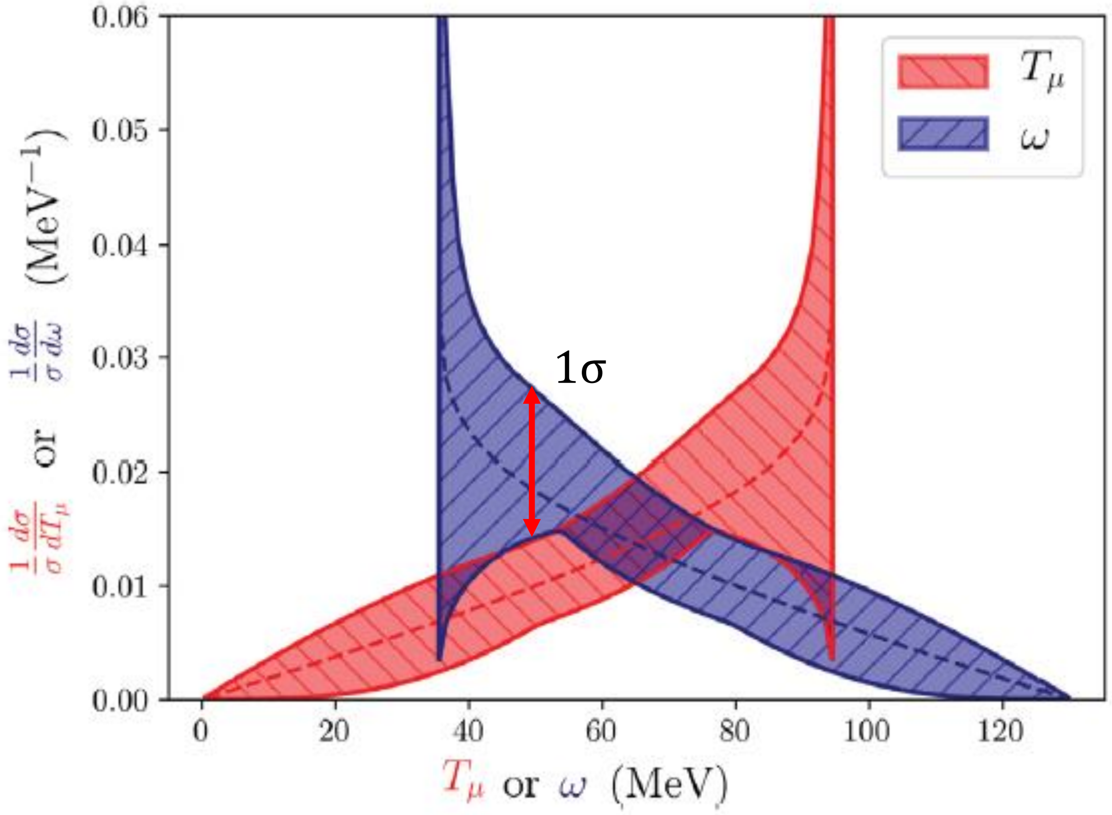
- In accelerator experiments, most neutrinos are produced by the decay-in-flight(DIF) of kaons and pions and three-body beta decay, which creates a wide energy spectrum and makes difficult.
- Different from neutrinos produced in DIF which have a broad energy spectrum, kaon decay-at-rest(KDAR) neutrinos are produced with **236 MeV monoenergetic**.
- Experiments on DIF neutrinos, which have high energy range in GeV scale, have been limited. But KDAR neutrinos have with 236 MeV monoenergetic with an **energy transfer of about 130 MeV**, which includes the region reaction of neutrino process in supernovae.
- The MiniBooNE experiment performed $^{12}\mathrm{C}-\nu_{\mu}$ differential cross section measurements using KDAR neutrinos, and the J-PARC Spallation Neutron Source (JSNS^2) experiment is currently conducting high intensity KDAR neutrinos-based measurements.



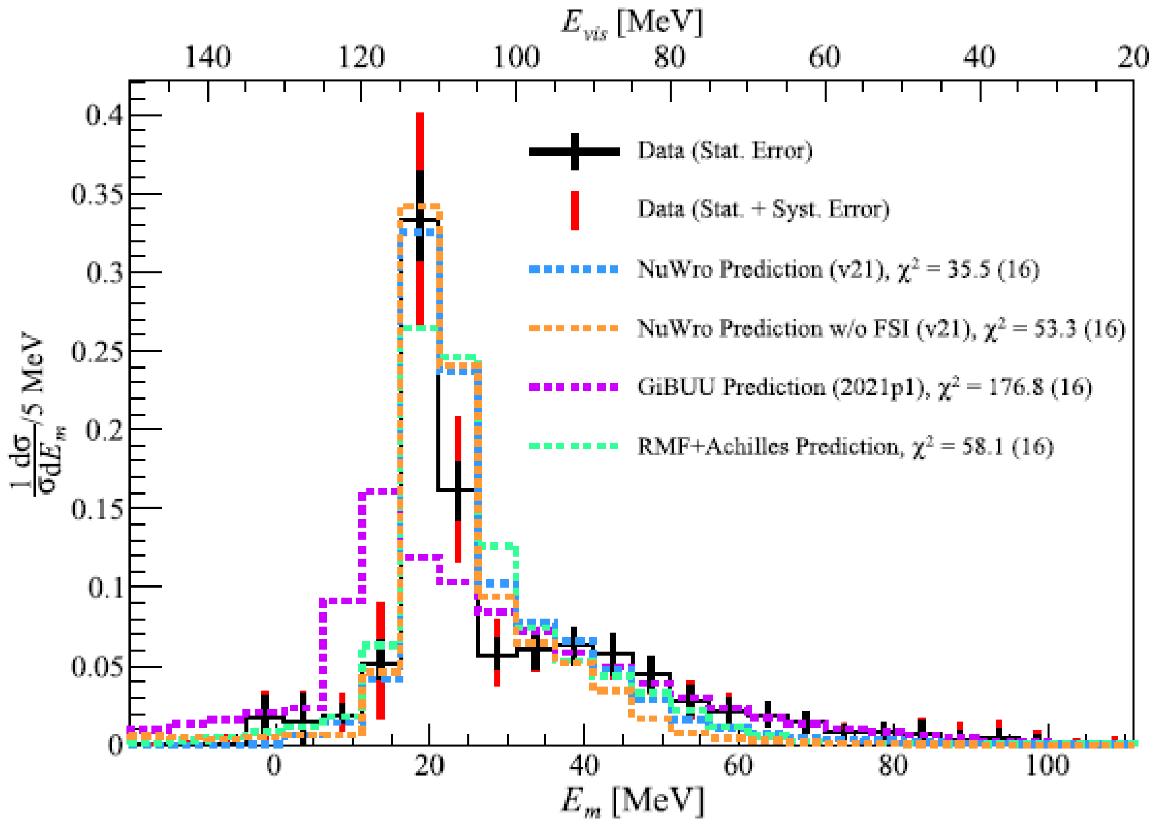
[2] J. Spitz, Physical Review D 89, 073007 (2014)



Introduction



- The MiniBooNE experiment $^{12}\mathrm{C}-\nu_\mu$



- JSNS^2 experiment $^{12}\mathrm{C}-\nu_\mu$

$$E_m \equiv \omega - \sum T_p = E_{\nu_\mu} - m_\mu + [m_n - m_p] - E_{vis}$$

[2] A. A. Aguilar-Arevalo *et al* ,*Phys. Rev. Lett.* **120** 141802 (2018)
[3] E. Marzec *et al, Phys. Rev. Lett.* **134** 081801 (2025)

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Motivation

- When probing nuclear structure with electrons, only protons can typically be observed because the interaction proceeds via the electromagnetic interaction.
- Neutrons are electrically neutral, they generally do not interact with electrons, leading to limitations in observing nuclear structure.
- When using neutrinos for probing, the weak interaction allows both neutrons and protons to be observed.
- Unlike electromagnetic interactions, weak interactions allow us to probe nuclear structure not only through vector currents but also through axial currents.
- Neutrino-induced reactions have been known to play important roles on the nucleosynthesis in core collapsing supernovae explosions.
- Most of approaches for the neutrino-induced reactions exploited the shell model (SM) or the quasi-particle random phase approximation (QRPA). The shell model has limitation in heavy nuclei because of tremendous increase of configuration mixings.
- Unlike previous studies that primarily focused on total cross-section or flux-averaged observations, we aimed to explicitly analyze the spin-parity dependence of the nuclear response at fixed neutrino energy.

Formalism

Quasiparticle Random Phase Approximation(QRPA)

- Quasiparticle phonon creation operator

$$Q^{\dagger,m}_{JM} = \sum_{kl\mu'\nu'} \left[X^{m}_{(k\mu'l\nu'J)} C^{\dagger}(k\mu'l\nu'JM) - Y^{m}_{(k\mu'l\nu'J)} \tilde{C}(k\mu'l\nu'JM) \right]$$

- Quasiparticle phonon annihilation operator

$$Q^{m}_{JM} = \sum_{kl\mu'\nu'} \left[X^{m*}_{(k\mu'l\nu'J)} C(k\mu'l\nu'JM) - Y^{m*}_{(k\mu'l\nu'J)} \tilde{C}^{\dagger}(k\mu'l\nu'JM) \right]$$

k, l, J, M (Roman letters) : single particle states
μ, ν (Greek letters) : isospin of quasiparticle

$X^{m}_{(k\mu'l\nu'J)}, Y^{m}_{(k\mu'l\nu'J)}$: Forward and backward amplitude

$C^{\dagger}(k\mu'l\nu'JM) = \sum_{m_k m_l} C^{JM}_{j_k m_k j_l m_l} a^{\dagger}_{l\nu'} a^{\dagger}_{k\mu'}$:Pair creation operator

$\tilde{C}(k\mu'l\nu'JM) = (-)^{J-M} C(k\mu'l\nu'J-M)$:Pair annihilation operator

$C^{JM}_{j_k m_k j_l m_l}$: Clebsch-Gordan coefficients

$a^{\dagger}_{l\nu}$: Quasiparticle creation operators

$$a^{\dagger}_{l\nu} = u_l c^{\dagger}_{l\nu} - v_l c_{\tilde{l}\nu}$$

amplitude   particle creation operator

$$\begin{pmatrix} A & B \\ -B & -A \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = E \begin{pmatrix} X \\ Y \end{pmatrix}$$

$$\langle m; J \| \hat{\mathcal{O}}_J \| 0^+ \rangle = \sum_{ab} \left[X^{m}_{ab} \langle a \| \hat{\mathcal{O}}_\lambda \| b \rangle u_a v_b + Y^{m}_{ab} \langle b \| \hat{\mathcal{O}}_\lambda \| a \rangle u_b v_a \right]$$

[4] J. Suhonen, *From Nucleons to Nucleus*, (Springer, Berlin, 2007)

Formalism

- Charged current and Neutral current

  - **Charged current weak interaction**

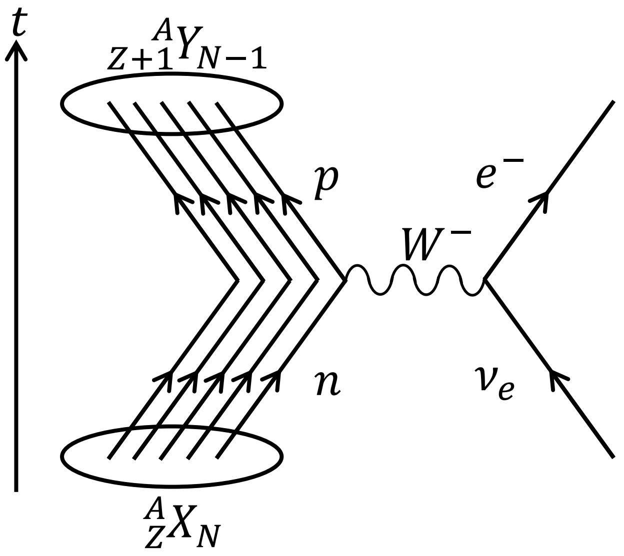
    Exchange of $\mathrm{W}^{\pm}$ boson
    Change flavor of quark, lepton and electrical charge

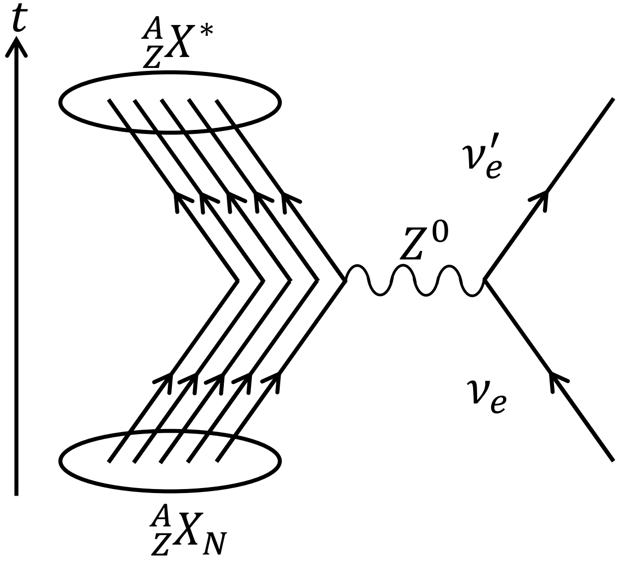
    ${}^{12}_{6}\mathrm{C}_{6}(\nu_e, e^-)\,{}^{12}_{7}\mathrm{N}_{5}$

  - **Neutral current weak interaction**

    Exchange of Z^{0} boson
    Transfer momentum, spin and energy

    ${}^{12}_{6}\mathrm{C}_{6}(\nu_e, \nu_e')\,{}^{12}_{6}\mathrm{C}_{6}^{*}$





Formalism

- Electromagnetic and weak current operator

  - **Electromagnetic current operator**

    Vector form factor

$$J_{\text{EM}} = V_3^{\mu} + V_0^{\mu}$$

$$F_V(Q^2) = \frac{g_V}{(1+Q^2/M_A^2)^2}$$

$$F_A(Q^2) = \frac{g_A}{(1+Q^2/M_A^2)^2}$$

  - **Weak current operator**

    Vector, axial and pseudo scalar form factors

$$F_P(Q^2) = \frac{2Mg_A}{m_\pi^2 - Q^2}$$

$$J_{\text{weak}}^{\mu} = J_V^{\mu} - J_A^{\mu} = \gamma^{\mu} F_1^{\text{V}}(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M_N} F_2^V(Q^2) + \gamma^{\mu}\gamma^5 F_A(Q^2) + \frac{q^{\mu}\gamma^5}{2M_N} F_P(Q^2)$$

$$g_A = 1.27$$
$$M_A = 855MeV$$

[5] Myung-Ki Cheoun *et al* 2010 *J. Phys. G: Nucl. Part. Phys.* **37** 055101

Formalism

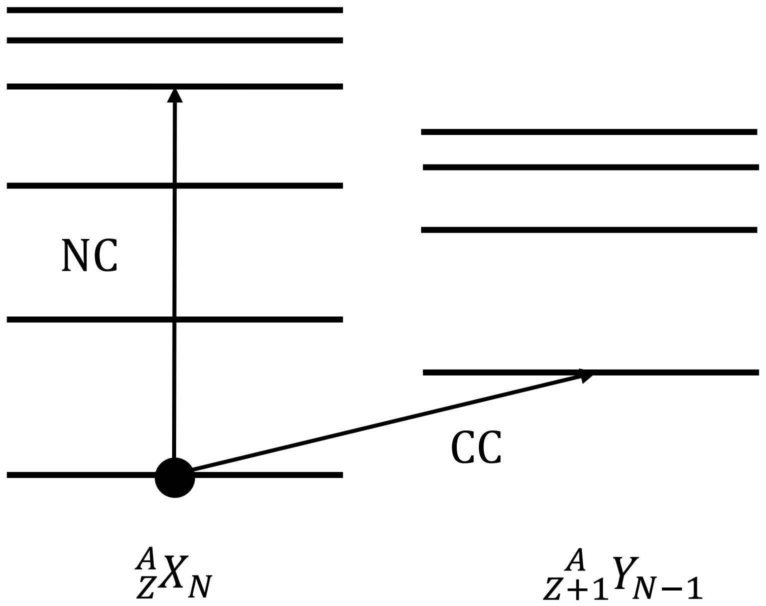
- Cross section

Coulomb

Longitudinal

$$\begin{aligned}\left(\frac{d\sigma_\nu}{d\Omega}\right)_{(\nu/\bar{\nu})} &= \frac{G_F^2\epsilon k}{\pi(2J_i+1)}\Bigg[\sum_{J=0}\Big\{(1+\vec{v}\cdot\vec{\beta})\left|\langle J_f\|\widehat{\mathcal{M}}_J\|J_i\rangle\right|^2 \\ &+\left(1-\vec{v}\cdot\vec{\beta}+2(\hat{v}\cdot\hat{q})(\hat{q}\cdot\vec{\beta})\right)\left|\langle J_f\|\hat{\mathcal{L}}_J\|J_i\rangle\right|^2-\hat{q} \\ &\cdot(\hat{v}+\vec{\beta})2\mathrm{Re}\langle J_f\|\hat{\mathcal{L}}_J\|J_i\rangle\langle J_f\|\widehat{\mathcal{M}}_J\|J_i\rangle^*\Big\} \\ &+\sum_{J\geq1}\Big\{\left(1-(\hat{v}\cdot\hat{q})(\hat{q}\cdot\vec{\beta})\right)\left(\left|\langle J_f\|\hat{\mathcal{T}}_J^{el}\|J_i\rangle\right|^2+\left|\langle J_f\|\hat{\mathcal{T}}_J^{mag}\|J_i\rangle\right|^2\right) \\ &\pm\left(\hat{q}\cdot(\hat{v}-\vec{\beta})2\mathrm{Re}\left[\langle J_f\|\hat{\mathcal{T}}_J^{mag}\|J_i\rangle\langle J_f\|\hat{\mathcal{T}}_J^{el}\|J_i\rangle^*\right]\right)\Big\}\Bigg]\end{aligned}$$

Magnetic Electric



$\vec{v},\vec{k}$ = three − momenta of incident&final leptons

$\vec{q}=\vec{k}-\vec{v},\vec{\beta}=\vec{k}/\epsilon$ $\quad\epsilon$ = final lepton′s energy

[6] Vernon W. Hughes *et al*, *Muon Physics II weak interaction*, (Academic press, Las Vegas, 2021), Vol 2, Sect 4. pp. 138-151

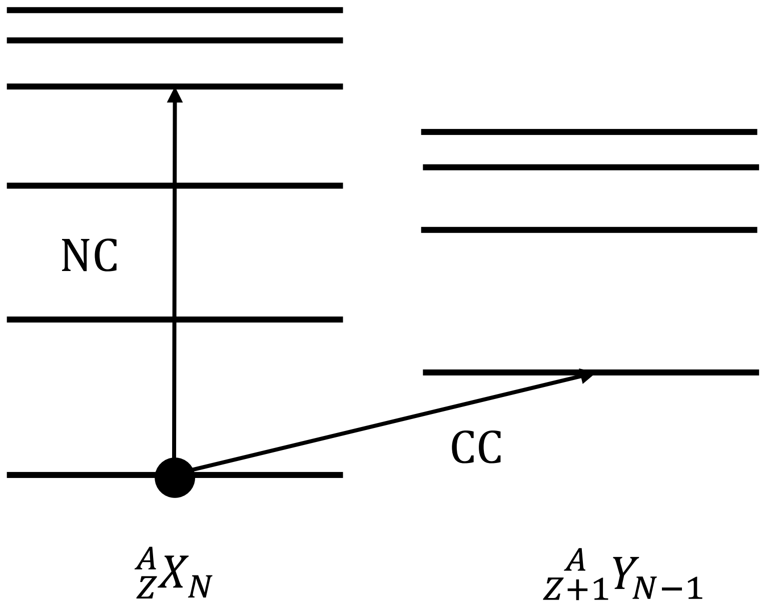
Formalism

- Strength function

Coulomb Longitudinal

$$\begin{aligned}\left(\frac{dB}{dT_\mu}\right)_{(\nu/\overline{\nu})} = \frac{1}{\pi(2J_i+1)}\Bigg[\sum_{J=0}\Big\{&|\langle J_f\|\widehat{\mathcal{M}}_J\|J_i\rangle|^2 + |\langle J_f\|\hat{\mathcal{L}}_J\|J_i\rangle|^2 \\ &- 2\mathrm{Re}\langle J_f\|\hat{\mathcal{L}}_J\|J_i\rangle\langle J_f\|\widehat{\mathcal{M}}_J\|J_i\rangle^*\Big\} \\ &+ \sum_{J\geq1}\Big\{\Big(|\langle J_f\|\hat{\mathcal{T}}_J^{el}\|J_i\rangle|^2 + |\langle J_f\|\hat{\mathcal{T}}_J^{mag}\|J_i\rangle|^2\Big) \\ &\pm 2\mathrm{Re}\big[\langle J_f\|\hat{\mathcal{T}}_J^{mag}\|J_i\rangle\langle J_f\|\hat{\mathcal{T}}_J^{el}\|J_i\rangle^*\big]\Big\}\Bigg]\end{aligned}$$

Magnetic Electric



$\vec{v}, \vec{k} = \text{three} - \text{momenta of incident\&final leptons}$
$\vec{q} = \vec{k} - \vec{v}, \vec{\beta} = \vec{k}/\epsilon \qquad \epsilon = \text{final lepton's energy}$

[6] Vernon W. Hughes *et al*, *Muon Physics II weak interaction*, (Academic press, Las Vegas, 2021), Vol 2, Sect 4. pp. 138-151

Formalism

- The weak field can be expanded in terms of multipole operators by using two basic operators

$$M_J^{M_J}(q\mathbf{x}) = j_J(q\mathbf{x})Y_J^{M_J}(\Omega_x) \qquad \mathbf{M}_{JL}^{M_J}(q\mathbf{x}) = j_J(q\mathbf{x})\mathbf{Y}_{JL1}^{M_J}(\Omega_x)$$

Vectorial spherical harmonics

$$\mathbf{Y}_{JL1}^{M_J}(\Omega_x) = \sum\nolimits_{m\lambda} \langle Lm1\lambda|(L1)JM_J\rangle Y_L^m(\Omega_x)\mathbf{e}_\lambda$$

- Coulomb, Longitudinal, Electric and Magnetic transition operators

$$\hat{\mathcal{M}}_{JM;TM_T}(q\mathbf{x}) = \left[F_1^{(T)}M_J^{M_J}(q\mathbf{x}) - i\frac{q}{M}\left[F_A^{(T)}\Omega_J^{M_T}(q\mathbf{x}) + \frac{F_A - \omega F_p^{(T)}}{2}\Sigma_J''^{M_J}(q\mathbf{x})\right]\right]I_T^{M_T}$$

$$\hat{\mathcal{L}}_{JM;TM_T}(q\mathbf{x}) = \left[\frac{-\omega}{q}F_1^{(T)}M_J^{M_J}(q\mathbf{x}) + i\left(F_A^{(T)} - \frac{q^2}{2M_N}F_P^{(T)}\right)\Sigma_J''^{M_J}(q\mathbf{x})\right]I_T^{M_T}$$

$$\hat{\mathcal{T}}^{el}_{JM;TM_T}(q\mathbf{x}) = \left[\frac{q}{M}\left[F_1^{(T)}\Delta_J'^{M_J}(q\mathbf{x}) + \frac{1}{2}\mu^{(T)}\Sigma_J^{M_J}(q\mathbf{x})\right] + iF_A^{(T)}\Sigma_J'^{M_J}(q\mathbf{x})\right]I_T^{M_T}$$

$$\hat{\mathcal{T}}^{mag}_{JM;TM_T}(q\mathbf{x}) = -i\frac{q}{M}\left[\left[F_1^{(T)}\Delta_J^{M_J}(q\mathbf{x}) - \frac{1}{2}\mu^{(T)}\Sigma_J''^{M_J}(q\mathbf{x})\right] + F_A^{(T)}\Sigma_J^{M_J}(q\mathbf{x})\right]I_T^{M_T}$$

Isoscalar and isovector

$T(= 0,1)$

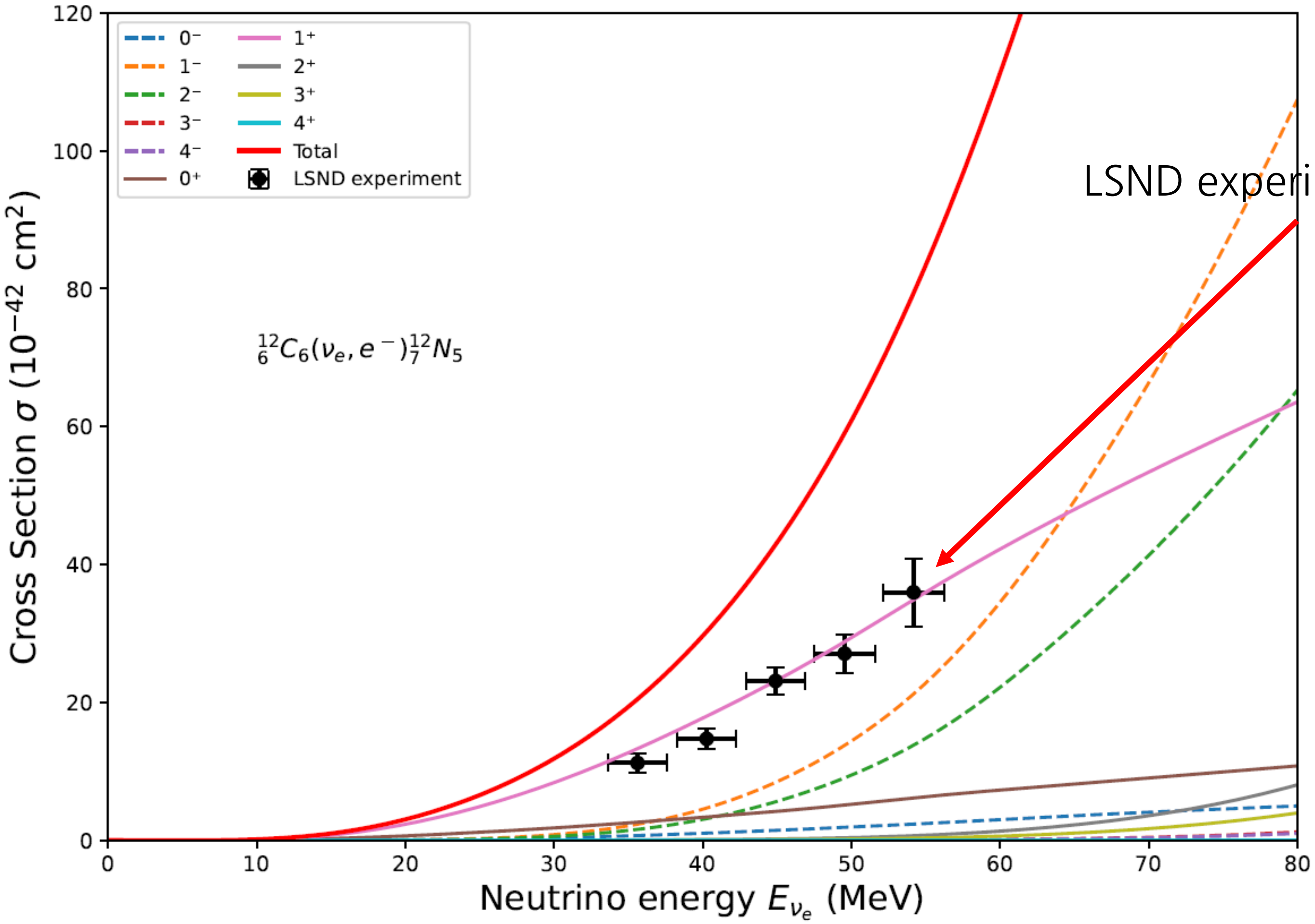
The eight relevant single particle operators

$$\left(M_J^{M_J}, \Omega_J^{M_J}, \Omega_J'^{M_J}, \Sigma_J^{M_J}, \Delta_J^{M_J}, \Sigma_J'^{M_J}, \Sigma_J''^{M_J}, \Delta_J'^{M_J}\right)$$

[8] T. W. Donnelly and W. C. Haxton, ATOMIC DATA AND NUCLEAR DATA 23, 103 (1979).

Result

- $^{12}_{6}\mathrm{C}-\nu_e$ charged current cross sections

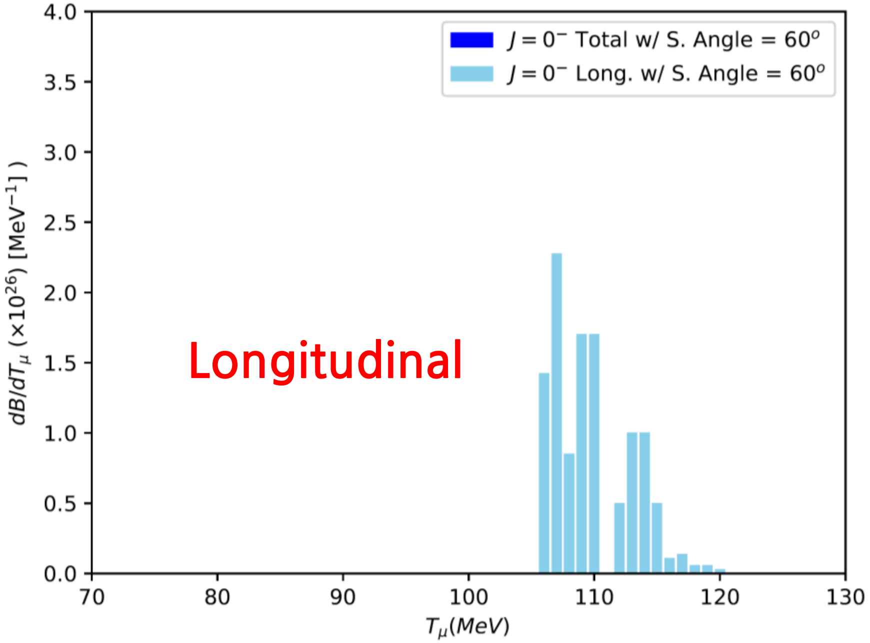


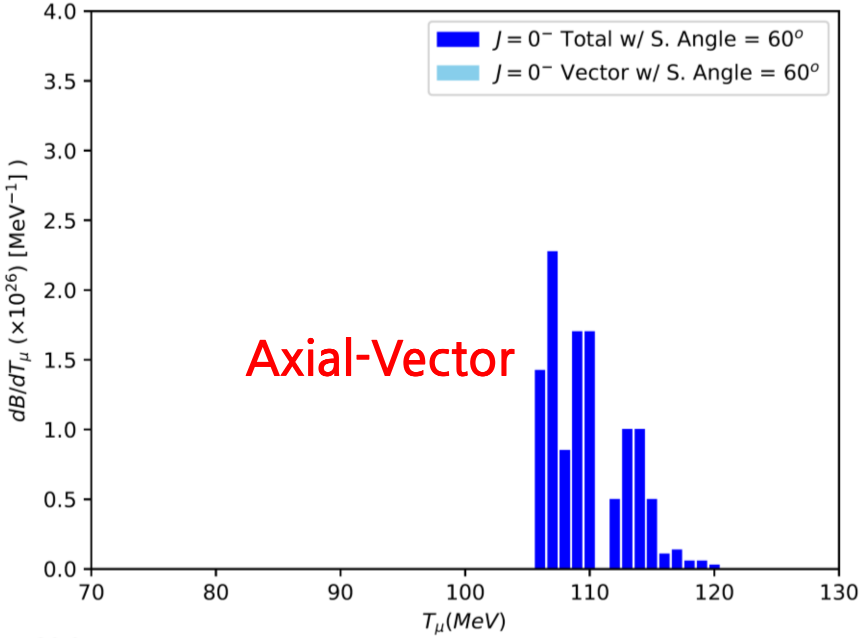
LSND experimental data $^{12}_{6}\mathrm{C}_6(\nu_e, e^-)^{12}_{7}\mathrm{N}_{5(1^+)}$

Charged current transition $^{12}_{6}\mathrm{C}_6(\nu_e, e^-)^{12}_{7}\mathrm{N}_5$

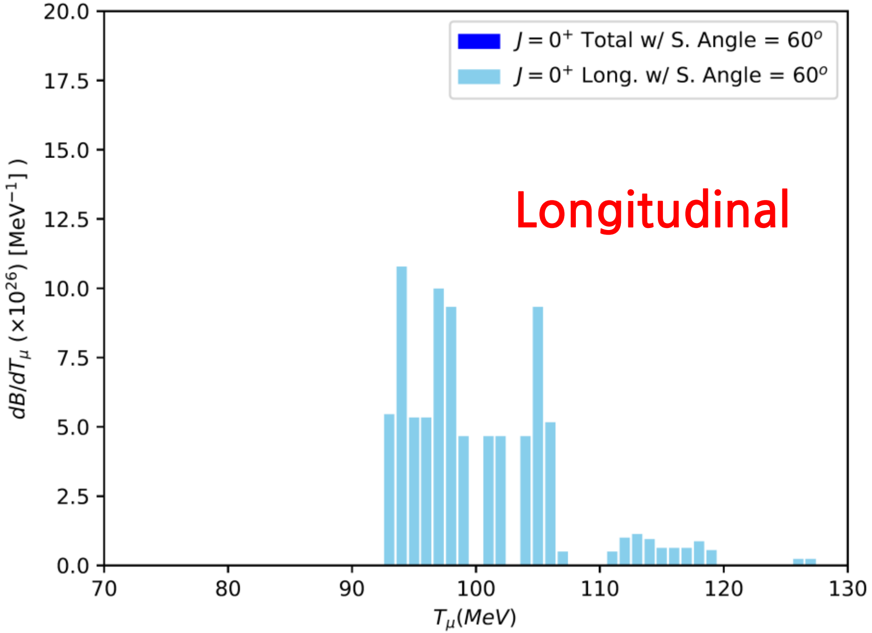
Result • $^{12}_{6}\mathrm{C}-\nu_{\mu}$ strength function $0^{\pm}$ w/ S. Angle $60°$

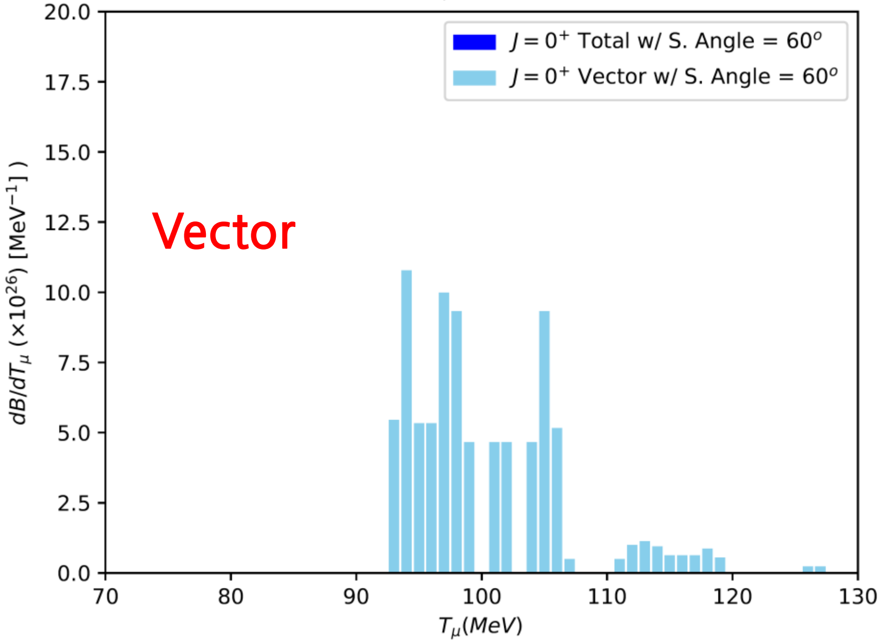
0^{-}





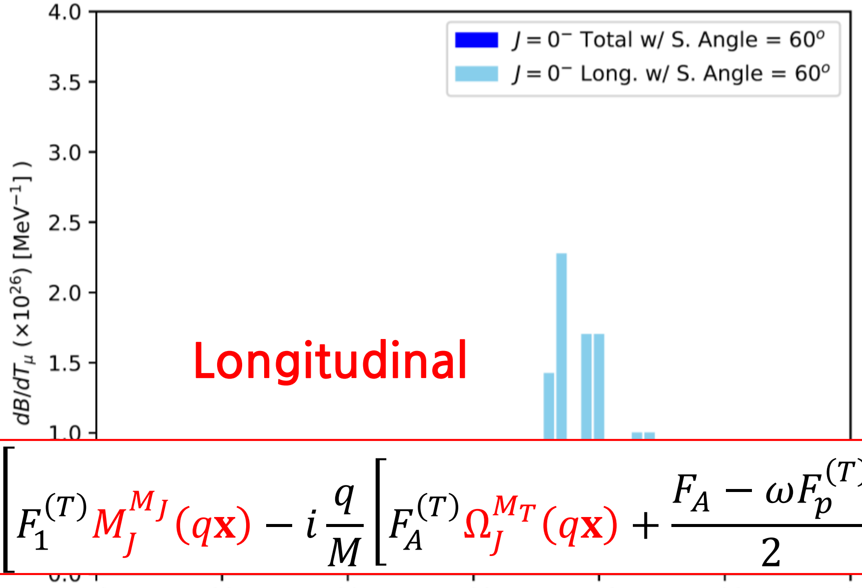
0^{+}



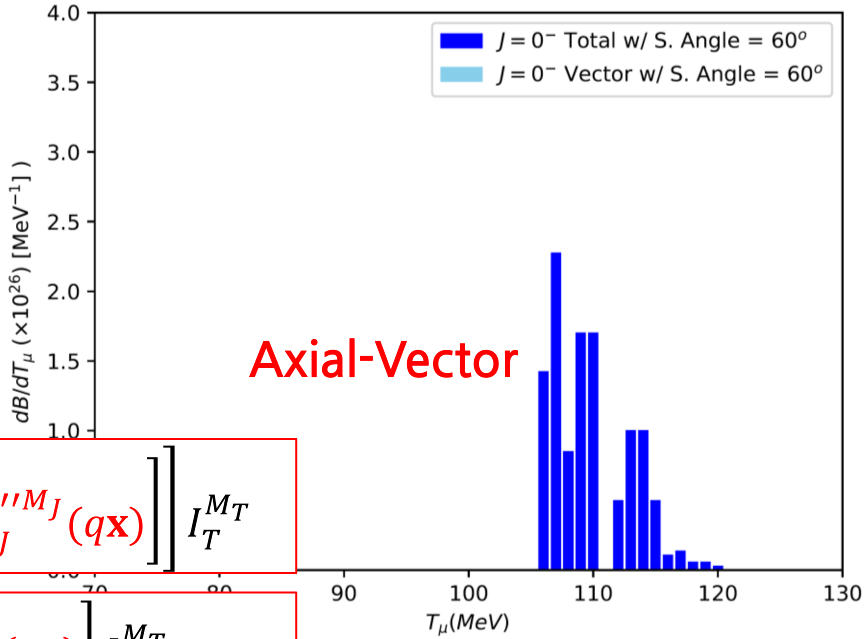


Result • ${}^{12}_{6}\mathrm{C} - \nu_\mu$ strength function $0^\pm$ w/ S. Angle 60°

0^-



Longitudinal

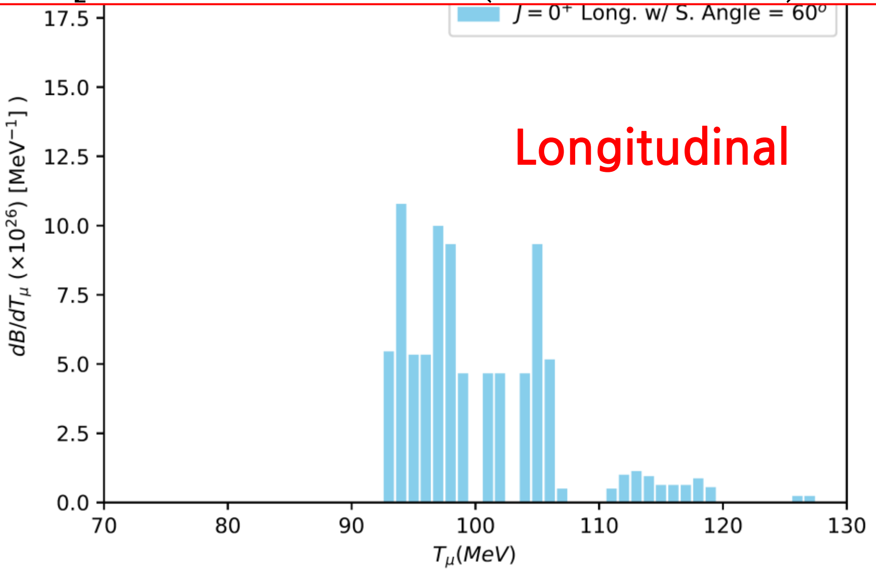


Axial-Vector

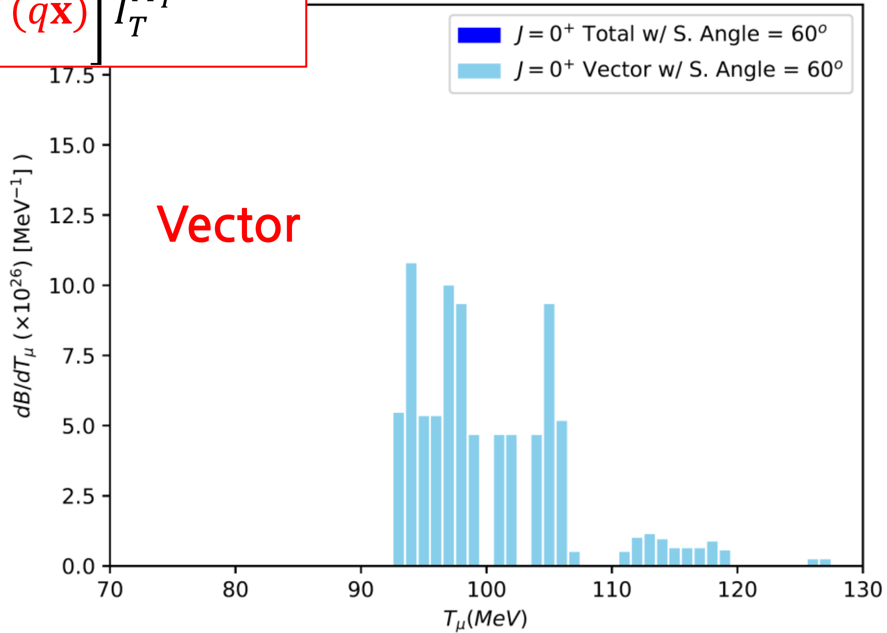
$$\widehat{\mathcal{M}}_{JM;TM_T}(q\mathbf{x}) = \left[F_1^{(T)} M_J^{M_J}(q\mathbf{x}) - i\,\frac{q}{M}\left[F_A^{(T)}\Omega_J^{M_T}(q\mathbf{x}) + \frac{F_A - \omega F_p^{(T)}}{2}\Sigma_J^{\prime\prime M_J}(q\mathbf{x})\right]\right] I_T^{M_T}$$

$$\hat{\mathcal{L}}_{JM;TM_T}(q\mathbf{x}) = \left[\frac{-\omega}{q}F_1^{(T)}M_J^{M_J}(q\mathbf{x}) + i\left(F_A^{(T)} - \frac{q^2}{2M_N}F_P^{(T)}\right)\Sigma_J^{\prime\prime M_J}(q\mathbf{x})\right] I_T^{M_T}$$

0^+



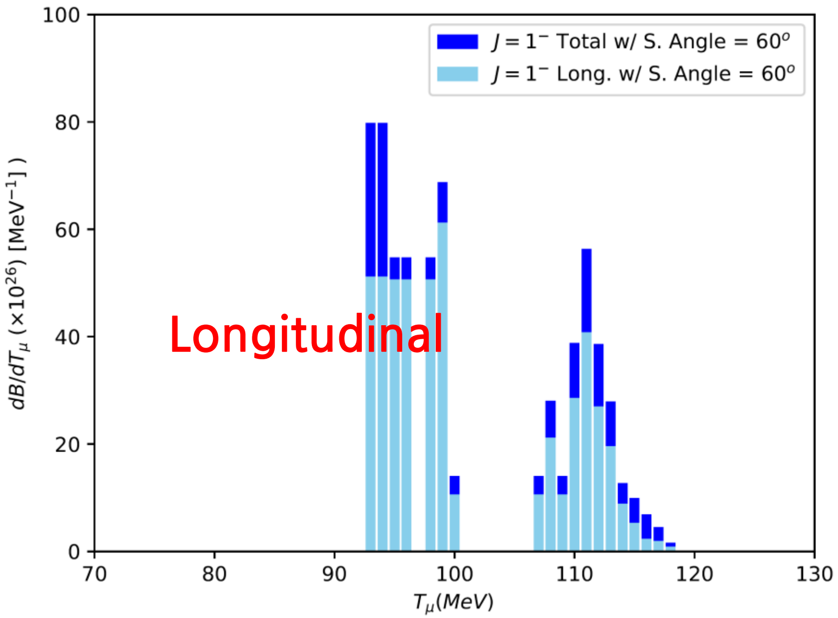
Longitudinal

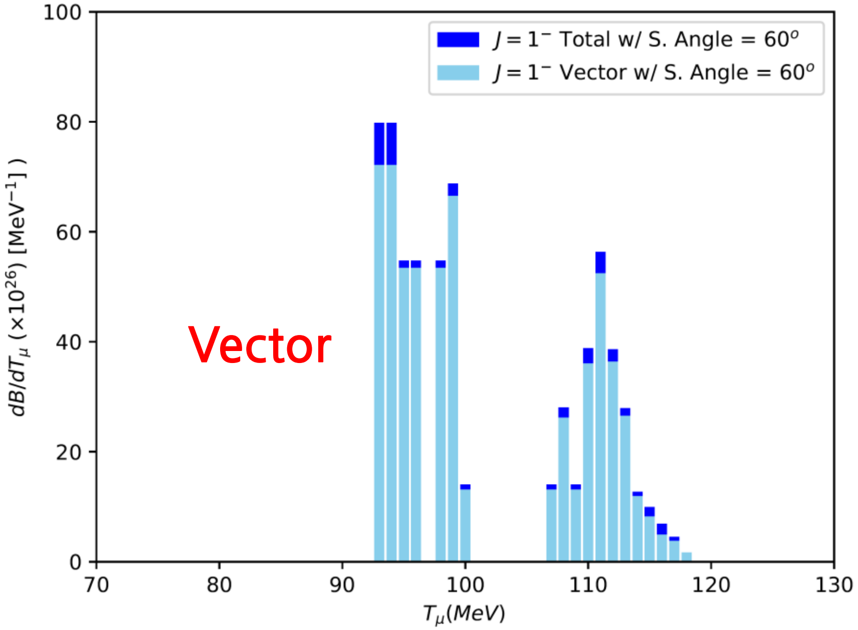


Vector

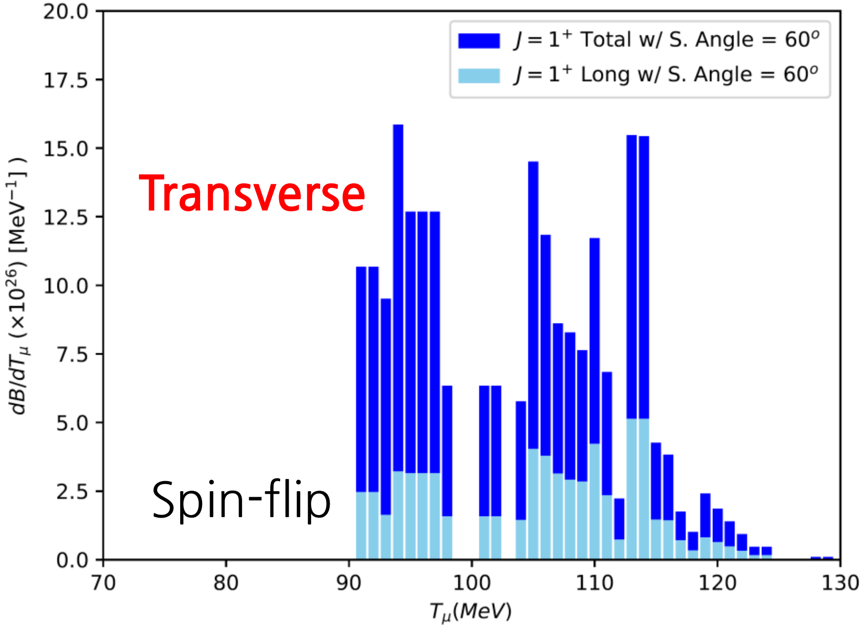
Result • ${}^{12}_{6}\mathrm{C} - \nu_\mu$ strength function $1^{\pm}$ w/ S. Angle $60°$

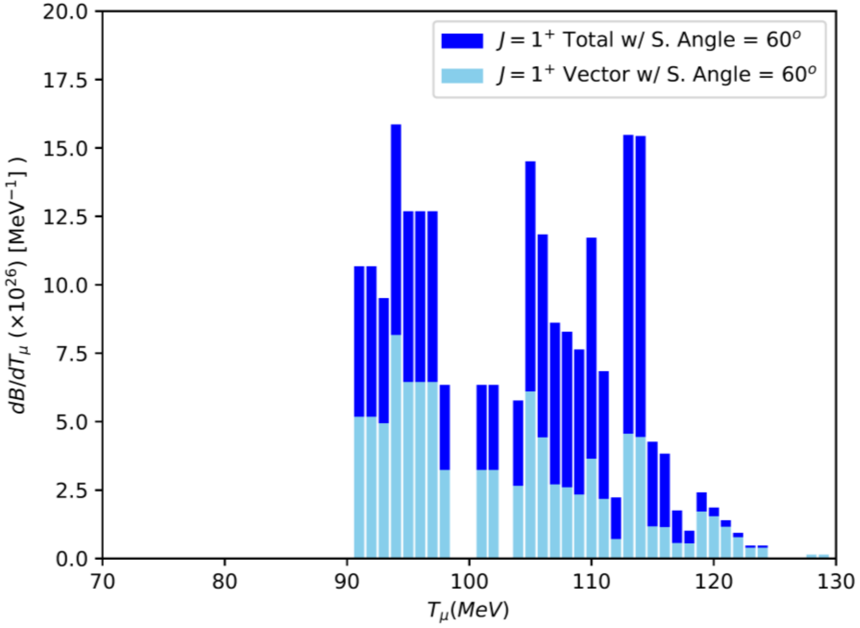
1^-





1^+

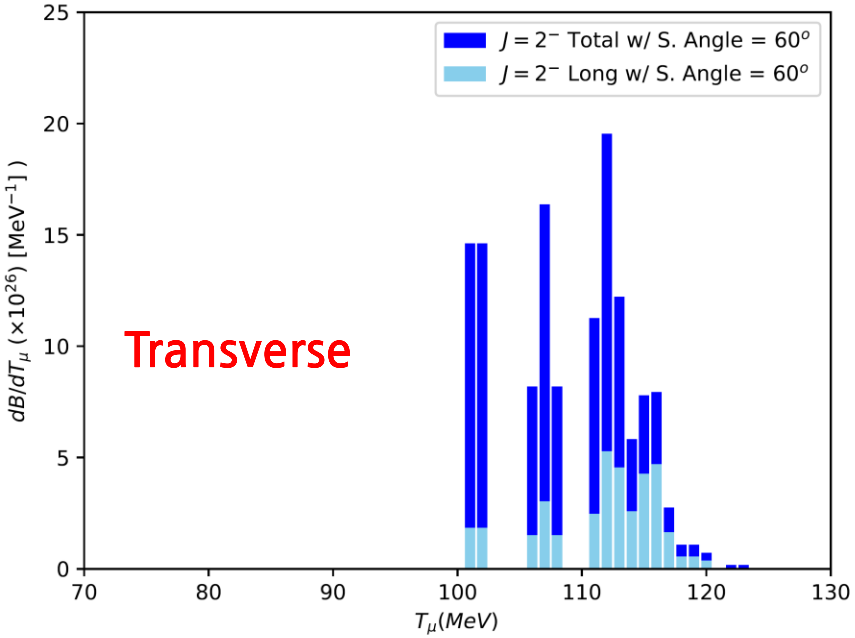


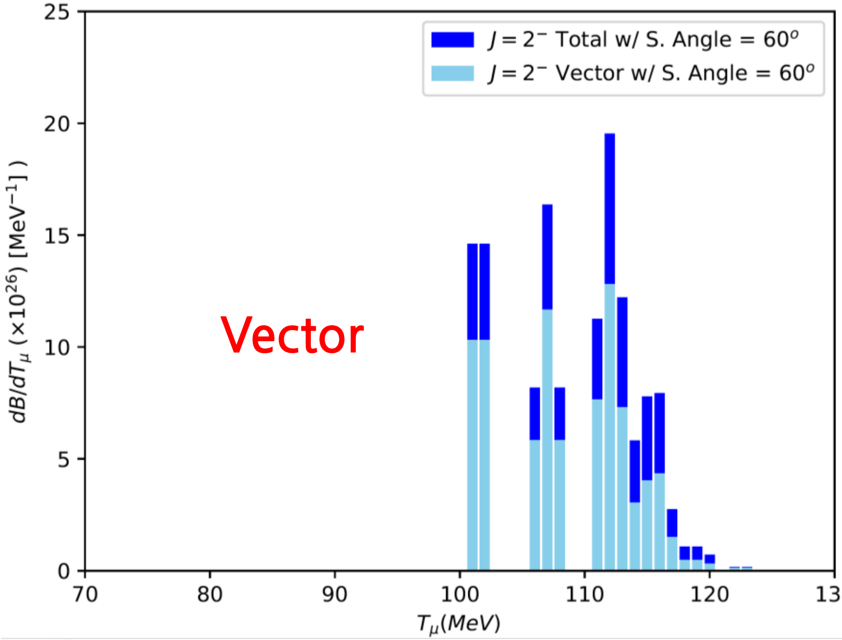


Result • ${}^{12}_{6}\mathrm{C}-\nu_{\mu}$ strength function $2^{\pm}$ w/ S. Angle $60°$

2^{-}

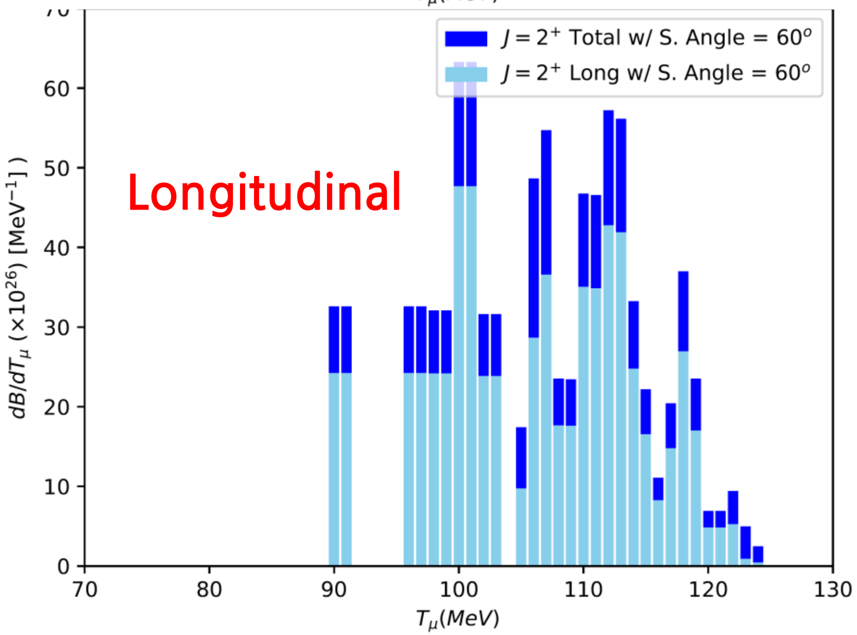
Spin-dipole

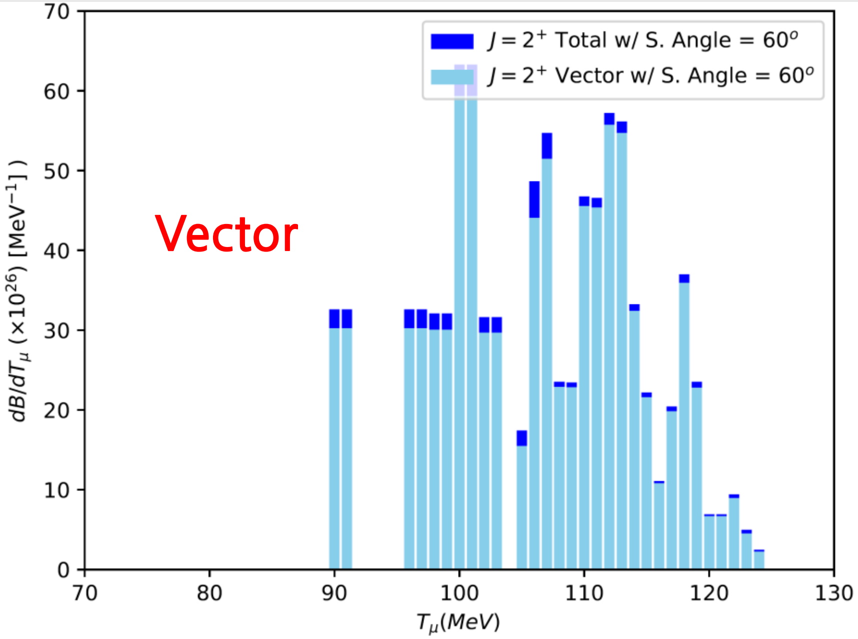




2^{+}

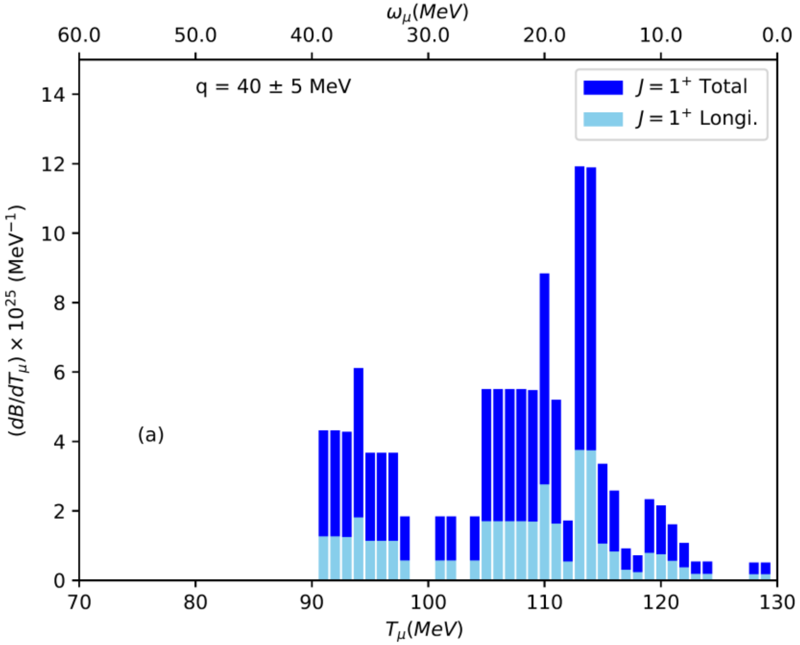
Collective-quadrupole

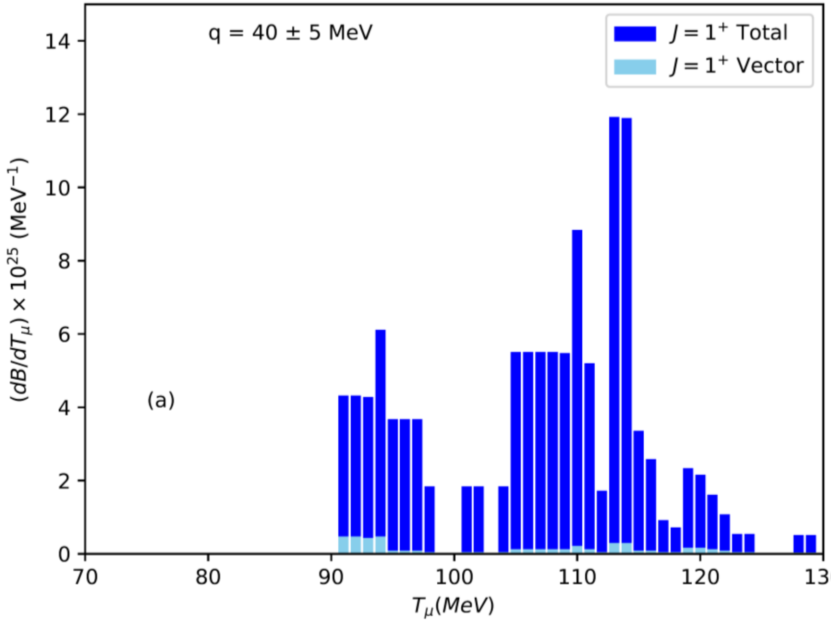




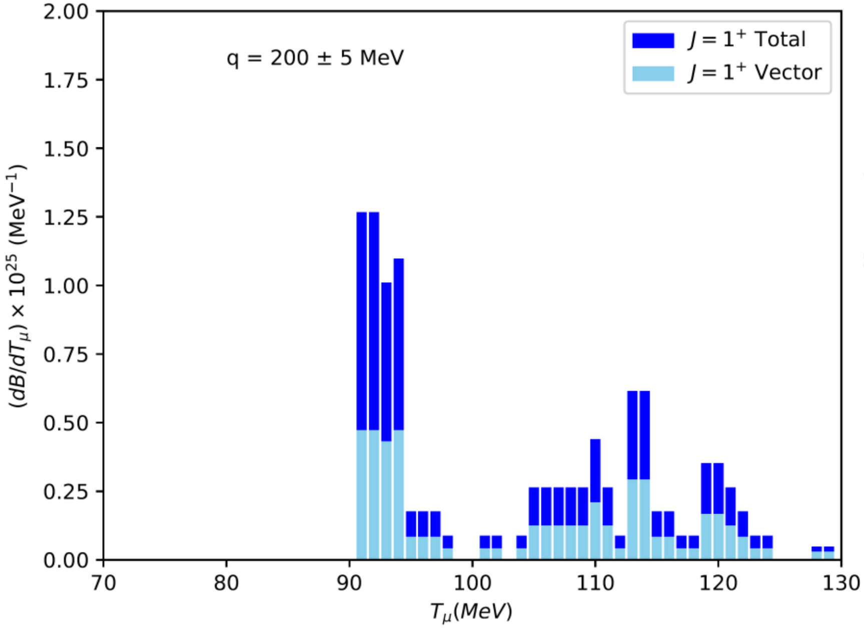
Result • ${}^{12}_{6}\mathrm{C}-\nu_{\mu}$ strength function $1^{\pm}$

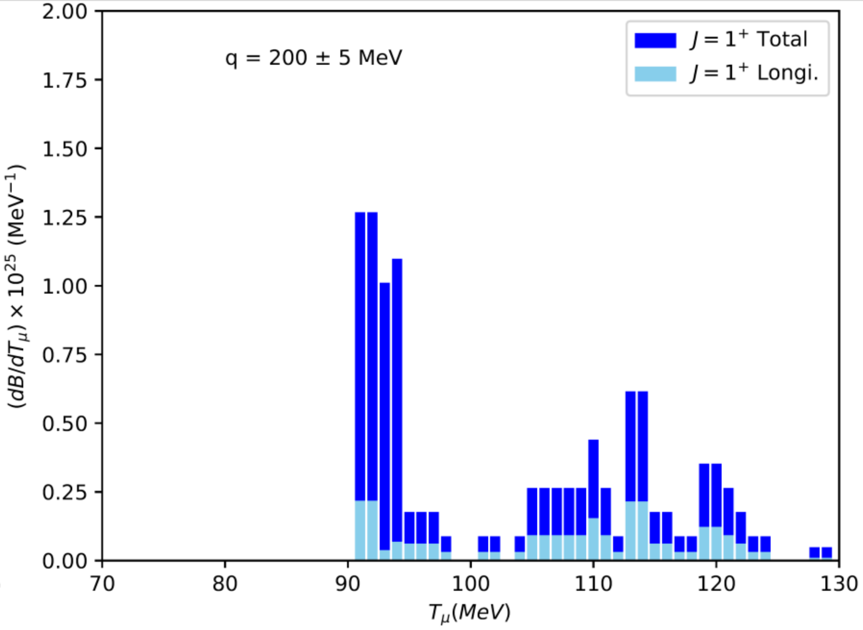
$q = 40\mathrm{MeV}$





$q = 200\mathrm{MeV}$





Summary

- By decomposing response functions into J^{π} states, this study confirmed that 0^{+} and 0^{-} transitions strictly follow vector and axial-vector selection rules, respectively, even at KDAR energies.

- While longitudinal responses dominate low-multipole transitions, transverse contributions and spin-flip dynamics become crucial as the multipole order increases.

- Using fixed-energy KDAR neutrinos allows for the precise tracking of nuclear structure and momentum transfer, revealing details often lost in flux-averaged measurements.

- These findings establish KDAR neutrinos as a powerful tool for probing the nuclear weak current, providing essential data to reduce systematic errors in future neutrino experiments.